International Journal of Innovative Research in Technology & Science (IJIRTS)

FIELD FUNCTIONS WITH SURFACE-DEFINED INFLUENTIAL RADII AND THEIR APPLICATIONS

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Abstract

In soft object modeling, soft objects are defined as a level surface of field functions, and a complex object can be connected smoothly through Boolean set blends. Because field functions are required to decrease to zero within the influential radius, union of soft objects is obtained easily by performing sum operation only, called soft blend. Besides, field functions with adjustable influential radii were also developed to adjust the size of the resulting set blends. However, the level surfaces of existing field functions always have similar shapes and sizes like the shape and size of soft object. As a result, unwanted blends and unnecessary bulges could be generated. To solve these problems, this paper proposes field functions with surface-defined influential radii, that is, it allow freely choosing implicit surfaces to define its inner and outer influential radii with any shapes. Thus, unwanted blend is removed by assigning blended primitives influential radii which does not contain the regions where unwanted blends lie; unnecessary bulge is avoided by assigning blended primitives influential radii which do not contain the region where unwanted bulges are located.

1. Introduction

In soft object modeling, primitive soft objects are defined as a 0.5 level-surface of field functions [1, 2, 3]. In order to create a more complex object, primitive soft objects, such as planes, super-ellipsoids, sweep surfaces, cylinders etc., can be connected smoothly through an automatically generated added or subtracted transition surface by Boolean set blends (union, intersection and difference). In the literature, boolean set blends include super-elliptical blends [4], blends with blending range parameters [5, 6] and blends of high-order smoothness [7]. Besides, sequential blends, i.e. a blend in blend, with affine-transformation are allowed and represented by a CSG (Constructive solid geometry) tree [8].

A field function is defined by a composition of a potential function and a distance function. Distance function [3, 9-14] determines the shape of a soft object. Potential function [1, 2, 11] enables the value of field function to decrease to zero in the influential radius and is zero outside the influential radius, and hence union of soft objects can be obtained by performing sum operation only, called soft blend [3] or by product operations only, called Perlin's boolean set blends [15]. To offer these two families of blends more shape controls, some filed functions were also developed. Field functions in [1] offer a softness parameter to adjust a transition's curvature; field functions [16, 17] offer blend range control by offering an inner and an outer radius parameters to adjust the inner and the outer influential radii of a field function through a transform of distance function. Locally restricted blends [16] allows each primitive to locally have different influential radii, like blend range, with the others by using a deformation function of field function and a graph of influential weights on one another.

In fact, field function in [16] adjusts all the inner or the outer radii in all directions with the same parameter value, so the shapes of level surfaces of a field function are all similar. On the contrary, this paper develops field functions with surface-defined influential radii, which extends field function with adjustable influential radii [16] by adding a freely chosen modulation surface in any shape. By using every influential radius of the modulation surface to individually vary the outer radius or the inner radius parameter in every direction, the shapes of the level surfaces of the proposed field function in added-material blend region or in subtracted-material blend region vary gradually from the primitive's shape to the chosen modulation surface. Thus, if soft objects are defined using the proposed field functions, soft blend and Perlin's and Ricci's set blends have the following applications by choosing suitable modulation surface:

- **Application 1**: When one primitive generates two blends at different places with other primitives, one can remove one blend but keep the other unchanged by choosing a modulation surface to obtain a new added-material blend region without including the region where the unwanted blend lies.
- **Application 2:** Bulges are eliminated in a union of intersecting and connecting super-elliptic cylinders and supertoroids by choosing modulation surfaces such that their added-material blend regions do not overlap where the unwanted bulge lies, even though these blends do not have range parameters to do bulge elimination by gradient-based methods [6, 18,19].

The remainder of this paper is organized as follows: Section 2 reviews implicit surfaces. Section 3 reviews soft object modeling and existing field functions. Section 4 presents field functions with surface-defined influential radii to solve the problems stated in Section 3. Section 5 demonstrates the applications of the proposed field functions. Conclusion is given in Section 6

2. Implicit surface modeling

2.1. Primitive implicit surface

In implicit surface modeling, an primitive implicit surface is represented as a level surface of a defining function $f_i(x,y,z): R^3 \rightarrow R$ by :

$$\{(x,y,z) \in R^3 \mid f_i(x,y,z)=C\},$$

where C is set a constant 0, 0.5 or 1. However, if the surface is viewed as an solid, it is redefined by a half space:

$$\{(x,y,z) \in R^3 \mid f_i(x,y,z) \leq C\} \text{ or } \{(x,y,z) \in R^3 \mid f_i(x,y,z) \geq C\}.$$

In the literature, according to the value of C, implicit surface modeling techniques have three categories:

- **Zero implicit surface** [19, 20, 21]:

$$\{(x,y,z) \in R^3 \mid f_i(x,y,z) \leq 0\},\ \{(x,y,z) \in R^3 \mid f_i(x,y,z) \geq 0\} \text{ or } \{(x,y,z) \in R^3 \mid f_i(x,y,z)=0\},$$

whose compliment is $\{(x,y,z) \in R^3 \mid -f_i(x,y,z) \leq 0\}$ or $-f_i(x,y,z) \geq 0\}$

- **Constructive geometry** [4, 6]:

$$\{(x,y,z) \in R^3 \mid f_i(x,y,z) \leq 1\} \text{ or } \{(x,y,z) \in R^3 \mid f_i(x,y,z)=1\},$$

whose compliment is $\{(x,y,z) \in R^3 \mid 1/f_i(x,y,z) \leq 1\}$.

- **Soft object modeling** [2, 5, 6, 6, 15, 18, 22]:

$$\{(x,y,z) \in R^3 \mid f_i(x,y,z) \geq 0.5\} \text{ or } \{(x,y,z) \in R^3 \mid f_i(x,y,z)=0.5\}.$$

whose compliment is $\{(x,y,z) \in R^3 \mid 1-f_i(x,y,z) \geq 0.5\}$.

In the following, a surface is denoted by $f_i(x,y,z)=C$, and a soft object by $f_i(x,y,z)=0.5$

2.2. Implicit blends

Furthermore, based on a blending operator $B_k(x_1,...,x_k)$: $R^k \rightarrow R$, a more complex implicit surface is constructed from k primitive implicit surfaces $f_i(x,y,z)=C$, $i=1,...,k$, and is defined using a blend $B_k(f_1,...,f_k)$ by

$$B_k(f_1(x,y,z),...,f_k(x,y,z))=C \text{ or}$$

$$\{(x,y,z) \in R^3 \mid B_k(f_1(x,y,z),...,f_k(x,y,z)) \leq \text{ or } \geq C\},$$

where $B_k(f_1,...,f_k)=C$ is called a blending surface. A blend connects primitive implicit surfaces smoothly by automatically generated added or subtracted transition surface which is tangent to blended primitives for eliminating sharp edges. Functionally, $B_k(f_1,...,f_k)=C$ has three kinds denoted as follows: (a). Union: $B_{Uk}(f_1,...,f_k)=C$, (b). Intersection: $B_{Ik}(f_1,...,f_k)=C$, and (c). Difference of f_1 from f_2,..., and f_k: $B_{Dk}(f_1, f_2, ...,f_k)=C$ as shown in Figure 1, which defines a wheel by sequential blends, depicted as a CSG tree.

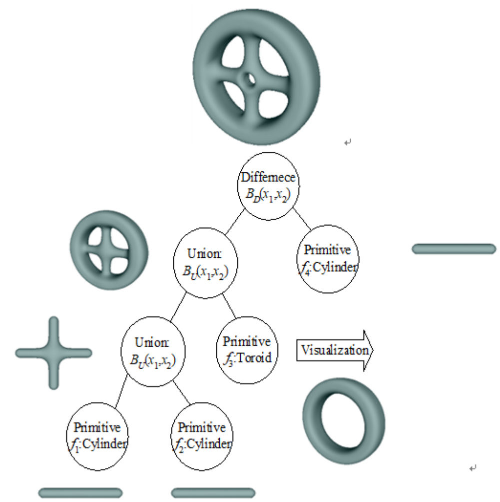

Figure 1. Wheel defined by $f(x,y,z)=B_{D2}(B_{U2}(B_{U2}(f_1,f_2),f_3),f_4)=C$ and depicted as a CSG tree.

3. Soft object modeling

3.1. Definition of soft objects and their blends

A primitive soft object is defined using a field function as a defining function by

$$f_i(x,y,z)=(P \circ d)(x,y,z) = P(d(x,y,z))=0.5, \qquad \textbf{(1)}$$

where $d(x,y,z): R^3 \rightarrow R_+$ is called distance function and $P(d): R_+ \rightarrow [0,1]$ is called potential function.

3.1.1. Distance function

A distance function $d(x,y,z)$ can be one of the structures: a point skeleton or 1D-3D skeletons [13]. As shown in Fig. 2, it is defined by

$$r/I_d = \|\overrightarrow{op}\|/\|\overrightarrow{oi}\|$$

where $r=(x^2+y^2+z^2)^{0.5}$ is the short distance from $p(x,y,z)$ to the skeleton and I_d is the influential radius specifying that the value of $(P \circ d)(x,y,z)$ is zero if $r \geq I_d$. I_d can be viewed as the distance from the point $\boldsymbol{o}$ on the skeleton where r is calculated to the surface $d(x,y,z)=1$ along the direction from o to p. In fact, a ray-linear $f(x,y,z)$ can be used as a distance function $d(x,y,z)$ because it can be reformulated as r/I_f. Existing distance functions include parallel planes, one-branch plane [14], super-ellipsoids [3, 11], super-quadrics [23], generalized distance functions [10], skeletal primitives [12], sweep objects [13] and spherical product functions [14].

3.1.2. Potential function

Potential function $P(d)$ is required to decreasing from 1 to 0 as d increases from 0 to 1 and satisfies the requirements $P(0)=1$, $P(0.5)=0.5$, $P'(0)=0$, $P'(1)=0$ and $P(d)=0$ for $d\geq 1$, all of which can be seen in Blanc's function $p=P_b(d)$ [1] written below:

$$P_b(d)=\begin{cases} 1-(3d^2)^2/(s+(4.5-4s)d^2) & d\leq 0.5 \\ (1-d^2)^2/(0.75-s+(1.5+4s)d^2) & 0.5<d\leq 1 \\ 0 & d>1 \end{cases} \quad (2)$$

where s is softness parameter.

The purposes of the requirements above are explained as below:

- $P(0.5)=0.5$ ensures that the shape of $P(d(x,y,z))=0.5$ is the same as $d(x,y,z)=0.5$ as in Figure 2. This tells that distance function determines the shape of a soft object.
- $P(0)=1$, $P(0.5)=0.5$ and $P(d)=0$ for $d\geq 1$ ensures that the value of $P(d(x,y,z))$ drops gradully to zero and is always zero when the value of r exceeds the influential radius I_d.
- Decreasing $P(d)$, $P'(0)=0$ and $P'(1)=0$.

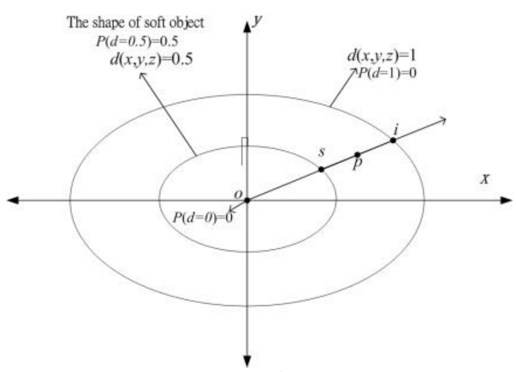

Figure. 2. Influential radius $I_r=\|\overrightarrow{oi}\|$ of a distance function $d(x,y,z)$, an ellipsoid, in x-y plane.

Because of these requirements, primitive soft objects $P(d(x,y,z))=0.5$, $i=1,...,k$, can be blended easily by sum and product operations only such as the blends below:

(a). Soft blend [2, 11]:

Union: $B_{SUk}(f_1,\ldots,f_k)=f_1(x,y,z)+...+f_k(x,y,z)=0.5.$ (3)

(b). Perlin' set blends [15]:

Union: $B_{PUk}(f_1,...,f_k)=1-(1-f_1)*(1-f_2)*(1-f_k)=0.5;$ (4)

Intersection: $B_{PIk}(f_1,...,f_k)=f_1*f_2*...*f_k=0.5;$ (5)

Difference: $B_{PUk}(f_1,f_2...,f_k)=f_1*(1-f_2)*...*(1-f_k)=0.5.$ (6)

In addition, some other blends with special functions on soft objects are listed below:

(a). Super-ellipsoidal blends [21], such as:

Union: $B_{RUk}(f_1,\ldots,f_k)=(f_1^{\,n}+\ldots+f_k^{\,n})^{1/n}=0.5.$ (7)

Intersection: $B_{RIk}(f_1,\ldots,f_k)=(f_1^{\,-n}+\ldots+f_k^{\,-n})^{-1/n}=0.5.$ (8)

where n is a curvature paremeter to adjust the curvature of the transition surface.

(b). Blends with blending range parameters [4, 15], which allow to adjust the size of the transition of a blend by varying the values of range parameters. These blends combined with gradient-based variable blending range [12, 15] can also eliminate unnecessary bulges. Figure 3 shows sequential unions of six intersecting cylinders by $B_{U2}(B_{U2}(B_{U2}(B_{U2}(B_{U2}(f_1,f_2),f_3),f_4)f_5),f_6)))=0.5$ without and with bulge eliminations, respectively.

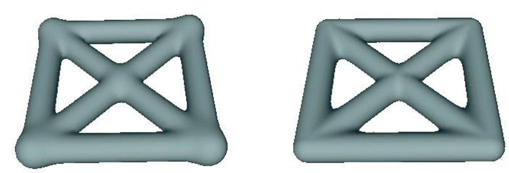

Figure 3. Sequential unions of six end-to-end connecting and intersecting cylinders defined with and without bulges.

3.2. Field functions with adjustable inner and outer influential radii

As shown in Figure 2, influential radius I_d can be divided into inner radius $\overrightarrow{os}=I_d/2$ and outer radius $\overrightarrow{si}=I_d/2$ and the inner radius and the outer radius influence the sizes of the subtracted-material (intersection) and the added-material (union) blending regions, respectively. The length of inner and outer radii indicate that a large soft object has a large blending region, a small one a small blending region.

To enable the adjustment of the lengths of inner and outer influential radii, field functions with adjustable inner and outer influential radii were proposed in [6, 17] and one of them is introduced as follows:

$$f_i(x,y,z)=(P\circ T_*\circ d_i)(x,y,z), \quad \textbf{(9)}$$

where $T_*(d)$ is a transform of $d_i(x,y,z)$, mapping $[0.5(1\text{-}w_2),$

0.5] to [0, 0.5] or [0.5, 0.5(1+w_1)] to [0.5, 1]. For example, $T_*(d)$ can be the normalization and affine transform $T_n(d)$ [6] given by

$$T_n(d)=\begin{cases} 1 & d > 0.5(1+w_1) \\ (d-0.5)/w_1+0.5 & 0.5 \le d \le 0.5(1+w_1) \\ N_n(d)=Ad^2+Bd+C & 0.5(1-w_2) \le d < 0.5 \\ 0 & \text{otherwise} \end{cases}, \quad (10)$$

and $w_2 \le 2w_1$, $w_2 \le 1$ and $w_1>0$; $A=2(1/w_1-1/w_2)/w_2$, $B=1/w_1-A$, $C=0.5-0.5B-0.25A$.

Thus, if soft objects are defined by $(P \circ T_n \circ d_i)(x,y,z)=0.5$, then adjusting w_2 and w_1 enables the inner and the outer radii change from $\overrightarrow{os} = I_d/2$ and $\overrightarrow{si} = I_d/2$ to $\overrightarrow{us}=w_2I_d/2$ and $\overrightarrow{st}=w_1I_d/2$, respectively and so subtracted-material and added-material blending regions become located in $0.5(1-w_2) \le d_i(x,y,z) \le 0.5$ and $0.5 \le d_i(x,y,z) \le 0.5(1+w_1)$, $i=1, \dots, k$, respectively, as shown in Figure 4.

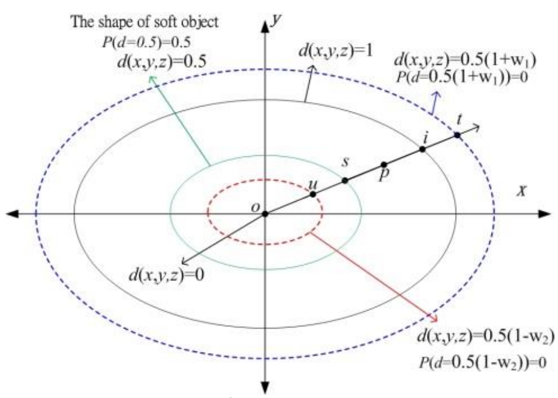

Figure 4. Inner radius $\overrightarrow{us}=w_2I_d/2$ and outer radius $\overrightarrow{st}=w_1I_d/2$ of $(P \circ T_* \circ d)=0.5$ of a ellipsoid in x-y plane and its subtracted-material and added-material blending regions bounded by new ellipsoids in red and blue lines.

3.3. Shapes of the level surfaces of existing field functions

When soft objects are defined by $(P \circ T_* \circ d_i)(x,y,z)=0.5$, $i=1,\dots,k$, in Eq. (9), in soft blend or Ricci's or Perlin's set blends $B_k(f_1,\dots,f_k)=0.5$ in Eqs. (3)-(8) level surfaces $d_i(x,y,z)=l$, $0.5(1-w_2) \le l \le 0.5(1+w_i)$, are used to generate the transition. Because $d_i(x,y,z)$ is always ray-linear, level surfaces $d_i(x,y,z)=l$ enlarge proportionally in all directions and have similar shapes as l increases. To enable level surfaces $d_i(x,y,z)=l$ of $(P \circ T_* \circ d_i)$, $0.5(1-w_{i2}) \le l \le 0.5(1+w_{i1})$, to have shapes different from soft object $d_i(x,y,z)=0.5$, this paper proposes field functions with surface-defined influential radii, denoted by $(P \circ T_{a*} \circ d_i)$ and $(P \circ T_{s*} \circ d_i)$, which is extended from $(P \circ T_* \circ d_i)(x,y,z)$ in Eq. (9) by adding a freely chosen modulation surface $f_{ai}(x,y,z)=0.5$ or $f_{si}(x,y,z)=0.5$ in shapes different from the primitives. Thus, as shown in Figure 5, by means of modulating the inner or the outer influential radius parameter w_1 and w_2 of T_* for every point's calculation through the influential radii $0.5I_{si}$ and $0.5I_{ai}$ bounded by $f_{ai}(x,y,z)=0.5$ or $f_{si}(x,y,z)=0.5$ in every direction:

(1). The shapes of $(P \circ T_{a*} \circ d_i)(x,y,z)=l$ for l=0.5 to 0, enlarge gradually from $d_i(x,y,z)=0.5$ to $f_{ai}(x,y,z)=0.5$ and its added-material blend region becomes $\{(x,y,z)\in R^3 | d_i(x,y,z) \ge 0.5$ and $f_{ai}(x,y,z) \le 0.5\}$.

(2). The shapes of $(P \circ T_{s*} \circ d_i)(x,y,z)=l$ for l=0.5 to 1 shrink gradually from $d_i(x,y,z)=0.5$ to $f_{si}(x,y,z)=0.5$ and its subtracted-material blend region becomes $\{(x,y,z)\in R^3 | f_{si}(x,y,z) \ge 0.5$ and $d_i(x,y,z) \le 0.5\}$.

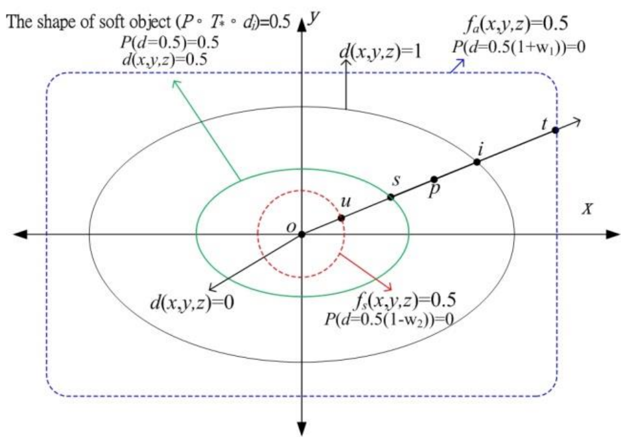

Figure 5. Inner radius $\overrightarrow{os}=I_d/2$ and outer radius $\overrightarrow{si}=w_1I_d/2$ of $(P_b \circ T_n \circ d)=0.5$, $w_1=w_2=1$, of a ellipsoid in x-y plane become $\overrightarrow{us}=0.5(I_d-I_{si})$ and $\overrightarrow{st}=0.5I_{ai}$ by varying w_1 and w_2 of T_n via chosen modulation surfaces sphere $f_s(x,y,z)=0.5$ and super-ellipsoid $f_a(x,y,z)=0.5$ in red and blue lines. The added-material blending region of $P \circ T_{an} \circ d)=0.5$ become bounded by $d(x,y,z)=0.5$ and $f_a(x,y,z)=0.5$. The subtracted-material blending region of $(P \circ T_{sn} \circ d)=0.5$ become bounded by $f_s(x,y,z)=0.5$ and $d(x,y,z)=0.5$.

Thus, due to the shape change of the level surfaces in the blending region controlled by modulation surfaces, soft blend and Perlin's and Ricci's set blends on soft object $(P \circ T_{s*} \circ d_i)(x,y,z)=0.5$ have the following new functions by choosing suitable modulation surfaces $f_{ai}(x,y,z)=0.5$ or $f_{si}(x,y,z)=0.5$:

- **Application 1**: When one primitive has two blends with other primitives at two places, one can avoid one blend unwanted but keep the other unchanged by choosing a modulation surface to define a new added-material blend region without containing the region where the unwanted blend lies.

- **Application 2:** Bulge elimination for a union of intersecting and connecting super-elliptic cylinders and super-toroids is achieved by choosing modulation surfaces such that their union blend regions do not overlap where the unwanted bulge lies.

4. Field functions with surface-defined influential radii

Let $T_*(d)$ denotes a transform of distance function $d_i(x,y,z)$ like T_n in Eq. (9). Extended from $(P \circ T_* \circ d_i)(x,y,z)$, field functions with surface-defined influential radii, denoted by $(P \circ T_{s*} \circ d_i)(x,y,z)$, for union, intersection and difference are presented in Sections 4.1-3.

4.1. Field functions with surface-defined outer radius for a union blend $B_{Uk}(f_1,\ldots,f_k)$=0.5

For each primitive f_i in B_{Uk}, define a field function as described below:
Step 1: Choose a surface $f_{ai}(x,y,z)$=0.5 that satisfy the conditions: (a). $f_{ai}(x,y,z)$ is ray-linear, (b). it and $d_i(x,y,z)$ both has the same structure: point, or 1-3D skeleton, and (c). $d_i(x,y,z)\leq 0.5$ is included within $f_{ai}(x,y,z) \leq 0.5$.

Step 2: For making the added-material blending region $\{(x,y,z)\in R^3|f_i(x,y,z)\leq 0.5$ and $f_i(x,y,z)\geq 0\}$ of $\boldsymbol{f_i}$=0.5 become $\{(x,y,z)\in R^3|d_i(x,y,z)\geq 0.5$ and $f_{ai}(x,y,z)\leq 0.5\}$, vary the outer radius parameter w_1 of $\boldsymbol{f_i}$ by replacing $\boldsymbol{f_i}$ with $(P \circ T_{a*} \circ d_i)(x,y,z)$ written by

$$\boldsymbol{f_i} =(P \circ T_{a*} \circ d_i)(x,y,z)\ ,\ \ i=1,\ldots, \text{ or } k, \qquad \textbf{(11)}$$

where $T_{a*}(d)$ is $T_*(d)$ in Eq. (9) with $w_1= d_i(x,y,z)/f_{ai}(x,y,z)-1$,

$(P \circ T_{a*} \circ d_i)$'s new blending region is derived by substituting $d_i(x,y,z)/f_{ai}(x,y,z)-1$ for w_1 in $\{(x,y,z) \in R^3|d_i(x,y,z) \geq 0.5$ and $d_i(x,y,z)\leq 0.5(1+w_1)\}$ yields $\{(x,y,z)\in R|\ d_i(x,y,z)\geq 0.5$ and $d_i(x,y,z)\leq 0.5(d_i(x,y,z)/f_{ai}(x,y,z)-1+1)\}\equiv\{(x,y,z)\in R^3|d_i(x,y,z)\geq 0.5$ and $f_{ai}(x,y,z)\leq 0{,}5\}$, completing the proof. Prove it from influential radii I_d and I_{fa} of $d_i(x,y,z)$ and $f_{ai}(x,y,z)$. Let $d_i(x,y,z)$ be represented by r/I_d and $f_{ai}(x,y,z)$ by r/I_{fa}. Then for any point (x,y,z) its influential radius is $0.5(1+w_1)I_d$= $0.5(1+r/I_d/r/I_{fa}-1)I_d$=$0.5(1+I_{fa}/I_d-1)I_d$=$0.5I_{fa}$, bounded by $f_{ai}(x, y, z)$=0.5 and so completing the proof.

4.2. Field functions with surface-defined inner radius for an intersection blend $B_{Ik}(f_1,\ldots,f_k)$ =0.5

For each primitive in B_{Ik}, define a field function as described below:
Step 1: Choose a surface $f_{si}(x,y,z)$=0.5 that satisfy the conditions: (a). $f_{si}(x,y,z)$ is ray-linear, (b). it and $d_i(x,y,z)$ both have the same structure, and (c). surface $f_{si}(x,y,z)\leq 0.5$ is included within $d_i(x,y,z)\leq 0.5$.

Step 2: For making the subtracted-material blending region $\{(x,y,z)\in R^3|f_i(x,y,z)\geq 0.5$ and $f_i(x,y,z)\geq 1\}$ of $\boldsymbol{f_i}$=0.5 become $\{(x,y,z)\in R^3|d_i(x,y,z)\leq 0.5$ and $f_{si}(x,y,z)\geq 0.5\}$, vary the outer radius parameter w_2 of T_* in $\boldsymbol{f_i}$ by replacing $\boldsymbol{f_i}$ with $(P \circ T_{s*} \circ d_i)(x,y,z)$ written by

$$\boldsymbol{f_i} =(P \circ T_{s*} \circ d_i)(x,y,z)\ ,\ \ i=1,\ldots, \text{ or } k, \qquad \textbf{(12)}$$

where $T_{s*}(d)= T_*(d)$ where $w_2 = 1 - d_i(x,y,z)/f_{si}(x,y,z)$,

$(P \circ T_{s*} \circ d_i)$'s new blending region is derived by substituting $1-d_i(x,y,z)/f_{si}(x,y,z)$ for w_2 in $\{(x,y,z)\in R^3|\ d_i(x,y,z)\leq 0.5$ and $d_i(x,y,z)\geq 0.5(1-w_2)\}$ yields $\{(x,y,z)\in R|d_i(x,y,z)\leq 0.5$ and $d_i(x,y,z)\geq 0.5(1-(1-d_i(x,y,z)/f_{si}(x,y,z)))\} \equiv \{(x,y,z)\in R^3|d_i(x,y,z)\geq 0.5$ and $f_{si}(x,y,z)\geq 0{,}5\}$, completing the proof. Prove it from influential radii I_d and I_{fs} the same as in Eq. (11). Then for any point (x,y,z) its influential radius is $0.5(1-w_2)I_d$=$0.5(1-(1-r/I_d/r/I_{fs}))I_d$=$0.5(I_{fs}/I_d)I_d$=$0.5I_{fs}$, bounded by $f_{si}(x,y,z)$=0.5 and so completing the proof.

For example, Figure 6(a) display a soft object of a elliptic cylinder $(P_b \circ d_i)$=0.5 and $d_i=(((|x/10|^2+(y/6|^2)^{1/2})^{10}+|z/25|^{10})^{1/10}$, whose level surfaces $(P_b \circ d_i)=l$, l=0.5, 0.35, 0.15, 0.01, are always similar as in Figure 6(b). However, if the soft object is defined by $(P_b \circ T_{an} \circ d_i)$=0.5 in Eq. (11) possessing $f_{ai}(x,y,z)=(|x/24|^{10} +(y/15|^{10}+|z/50|^{10})^{1/10}$ shown in Figure 6(a) instead, then level surfaces $(P_b \circ T_{an} \circ d_i)=l$, l=0.5 0.35, 0.15, 0.01, enlarge from $d_i(x,y,z)$=0.5 to $f_{ai}(x,y,z)$=0.5 as in Figure 6(d); if the soft object is defined by $(P_b \circ T_{s*} \circ d_i)$=0.5 in Eq. (12) with $f_{si}(x,y,z)=(((|x/4|^2+(y/4|^2)^{1/2})^{10}+|z/12.5|^{10})^{1/10}$ shown in Figure 6(e) instead, then level surfaces $(P_b \circ T_{s*} \circ d_i)=l$, l=0.5, 0.65, 0.85, 0.99, shrink from $d_i(x,y,z)$ =0.5 to $f_{si}(x,y,z)$=0.5 as in Figure 6(f).

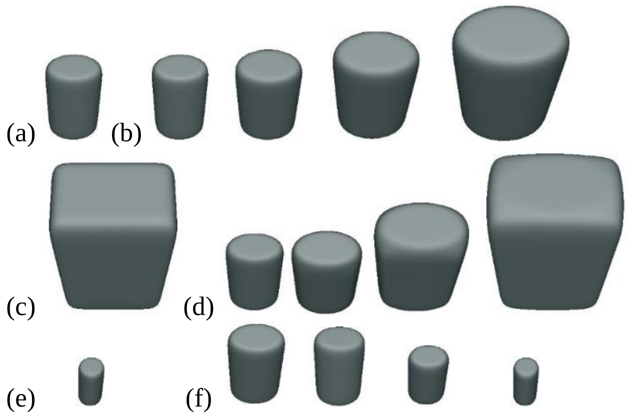

Figure 6. (a). soft object of an elliptic cylinder ($P_b \circ d_i$)=0.5. (b). Level surfaces ($P_b \circ d_i$)=l, l=0.5, 0.35, 0.15, 0.01. (c). Modulation surface $f_{ai}(x,y,z)$=0.5 for ($P_b \circ T_{an} \circ d_i$)=0.5 in (e). (d). Level surfaces ($P_b \circ T_{an} \circ d_i$)=$l$, l=0.5, 0.35, 0.15, 0.01. (e). Modulation surface $f_{si}(x,y,z)$=0.5 for ($P_b \circ T_{sn} \circ d_i$)=0.5 in (f). (f). Level surfaces ($P_b \circ T_{sn} \circ d_i$)=$l$, l=0.5, 0.65, 0.85, 0.99.

4.3. Field functions with a surface-defined inner and an outer radius for a difference blend $B_{Dk}(f_1, f_2,\dots,f_k)$=0.5

In a difference blend $B_{Dk}(f_1, f_2,\dots,f_k)$=0.5, the transition is generated in the outer radii of primitives except f_1, so

- For subtracted primitive $\boldsymbol{f_1}$ in B_{Dk}, define a field function as stated in Eq. (12) such that $\boldsymbol{f_1}=(P \circ T_{s*} \circ d_1)(x,y,z)$ has a new subtracted-material blending region:

  $\{(x,y,z)\in R^3 | d_1(x,y,z)\le 0.5 \text{ and } f_{s1}(x,y,z)\ge 0,5\}$.

- For each of subtracting primitives f_i, i=2,…,k in B_{Dk}, define a field function as described in Eq. (11) such that $\boldsymbol{f_i}=(P \circ T_{a*} \circ d_i)(x,y,z)$ has a new added-material blending region

  $\{(x,y,z)\in R^3 | d_i(x,y,z)\ge 0.5 \text{ and } f_{ai}(x,y,z) \le 0.5\}$.

4.4. Transformation function $T_*(d)$ for ($P \circ T_{a*} \circ d_i)(x,y,z)$ and $(P \circ T_{s*} \circ d_i)(x,y,z)$

This section presents some transforms that are suitable for being used as $T_*(d)$ in Eqs. (11)-(12).

4.4.1. Transformation function $T_*(d)$ for obtaining a surface-defined outer radius in Eq. (11)

- Modulation by $T_n(d)$ in Eq. (10).

$T_n(d)$ satisfies the mapping requirement [0.5, 0.5(1+w_1)] to [0.5, 1], but it need to make sure that $w_2 \le 2w_1$ and $w_2 \le 1$ hold while varying w_1. Hence, when applied in Eq. (11) w_1 and w_2 of $T_n(d)$ need to be modified additionally as follows:

w_1 of $T_n(d) = d_i(x,y,z)/f_{ai}(x,y,z) - 1$ and

w_2 of $T_n(d)$ =1 for w_1>0.5 or w_2 of $T_n(d)$ =2w_1 for $w_1 \le 0.5$.

Varying w_2 by w_1 such that $w_2 \le 2w_1$ is important for performing bulge elimination because w_1 approaches to zero very often.

- Modulation by *log* function $T_l(d)$:

$T_l(d)$ defined below maps [0, 0.5(1+w_1)] to [0, 1] and 0.5 to 0.5:

$$T_l(d) = \begin{cases} 1 & d > 0.5(1+w_1) \\ (2md)^{log(0.5)/log(m)} & d \le 0.5(1+w_1) \end{cases}, \qquad (13)$$

where $ln(t)$ denotes $log(t)$ function and m=1/(1+w_1) and w_1>0. $T_l(d)$ can be applied in Eq. (11) to adjust the outer radius by varying w_1.

Soft objects $(P \circ T_{an} \circ d_i)(x,y,z)$=0.5 and $(P \circ T_{al} \circ d_i)(x,y,z)$=0.5 offer an added-material blending region: $\{(x,y,z)\in R | d_i(x,y,z)\ge 0.5 \text{ and } f_{ai}(x,y,z)\le 0.5\}$, and the latter one do not need to vary w_2 by w_1.

4.4.2. Transformation function $T_*(d)$ for obtaining a surface-defined inner radius in Eq. (12)

- $T_n(d)$ in Eq. (10).

$T_n(d)$ also satisfies the mapping requirement [0.5(1-w_2), 0.5] to [0, 0.5]. Hence, when applied in Eq. (12), to satisfy $w_2 \le 2w_1$ w_1, and w_2 of $T_n(d)$ can be set additionally by:

w_2 = 1-$d_i(x,y,z)/f_{si}(x,y,z)$ and w_1= w_2.

- Modulation by *log* function $T_r(d)$:

Transform $T_r(d)$, defined below, maps [0.5(1-w_2), 1] to [0, 1] and 0.5 to 0.5.

$$T_r(d) = \begin{cases} 0 & d < 0.5(1-w_2) \\ 1-(2m(1-d))^{log(0.5)/log(m)} & d \ge 0.5(1-w_2) \end{cases}, \qquad (14)$$

where m=1/(1+w_2) and 0<$w_2 \le 1$. $T_r(d)$ can be applied in Eq. (12) to adjust the inner radius by varying w_2.

Soft objects $(P \circ T_{sn} \circ d_i)(x,y,z)=0.5$ and $(P \circ T_{sr} \circ d_i)(x,y,z)=0.5$ offer a subtracted-material blending region $\{(x,y,z)\in R|\ f_{si}(x,y,z)\geq 0{,}5$ and $d_i(x,y,z)\leq 0.5\}$ without varying w_1.

4.5. Ray-linear $f_{ai}(x,y,z)$ and $f_{si}(x,y,z)$

Because $f_{ai}(x,y,z)$ and $f_{si}(x,y,z$ in Eqs. (11)-(12) need to be ray-linear, this section shows an easy way to create a ray-linear function. As stated in [6], if $B_k(x_1,\ldots,x_k)$ is ray-linear and primitives $f_1,\ldots,$ and f_k are ray-linear, then blend $B_k(f_1,\ldots,f_k)$ is ray-linear, too. So, one may ray-linear blending operators such as union and intersection with range parameters in [5, 6] and super-ellipsoidal union and intersection [4] in Eqs. (7)-(8), and then create a ray-linear $f_{ai}(x,y,z)$ and $f_{si}(x,y,z)$ by performing applying these operators $B_k(x_1,\ldots,x_k)$ on the following primitives:

- Primitives of parallel planes, denoted as $f(x,y,z)$, to obtain symmetrical modulation surface $B_k(f_{p1},\ldots,f_{pk})=0.5$.

$$f_p(x,y,z)=|[x,\ y,\ z]\bullet\vec{n}|/a.$$

- Primitives of one-branch plane, denoted as $f_a(x,y,z)$, to obtain asymmetrical modulation surface $B_k(f_{o1},\ldots,f_{ok})=0.5$.

$$f_o(x,y,z)=[[x,\ y,\ z]\bullet\vec{n}]_+/a,$$

where $[*]_+\equiv Max(*,0)$, $\vec{n}$ denotes a normal vector toward the plane and $\bullet$ means dot product.

Based on these above, some modulation surfaces that offer six parameters, t_1, t_2, t_3, t_4, t_5, t_6 and … to adjust the lengths of positive and negative x, y and z axes, respectively, are created using super-ellipsoidal intersection blends and presented below:

(1). Super-quadrics [23]:

$$d(x,y,z)=((|x/a_1|^{n1}+|y/a_2|^{n1})^{n2/n1}+|z/a_3|^{n2})^{1/n2}$$

and its modulation surface:

$$f(x,y,z)=(([x/(t_1a_1)]_+^{n1}+[-x/(t_2a_1)]_+^{n1}+[y/(t_3a_2)]_+^{n1}+[-y/(t_4a_2)]_+^{n1})^{n2/n1}+[z/(t_5a_3)]_+^{n2}+[-z/(t_6a_3)]_+^{n2})^{1/n2} \quad \textbf{(15)}$$

(2). Generalized distance function [10]:

$$d(x,y,z)=(\textstyle\sum_{i=1}^{k}\ (|[x,y,z]\bullet\vec{n}_i/a_i|^n))^{1/n}$$

and its modulation surface:

$$f(x,y,z)=$$
$$(\textstyle\sum_{i=1}^{k}\ ([[x,y,z]\bullet\vec{n}_i/(t_ia_i)]_+^{n}+[[-x,-y,-z]\bullet\vec{n}_i/(t_{2i}a_i)]_+^{n})^{1/n} \quad (16)$$

(3). Spherical cross-product function [14]:

Given two 2D functions $h(x,y)$ and $m(x,z)$ by

$$h(x,y)=(|x/a_1|^n+|y/a_2|^n)^{1/n} \text{ and } m(x,z)=(x^n+|z/a_3|^n)^{1/n},$$

then a cross-product function $d(x,y,z)$ is written by

$$d(x,y,z)=m(h(x,y),\ z),$$

which can also vary to generate super-elliptic cylinders (Line skeletons):

$$d(x,y,z)=m(h(x,y),\ Sign(z)[|z|-L]_+), \quad (17)$$

where $Sign(*)$ srands for 1 if $*>0$, otherwise -1, $d(x,y,z)=1$ has a cross-section $h(x,y)=1$ and his spine is along z axis and $2L$ long.

As for the modulation surfaces for Eq. (17), just replace $h(x,y)$ and $m(x,z)$ with:

$$h(x,y)=(([x/(t_1a_1)]_+^{n}+[-x/(t_2a_1)]_+^{n}+[y/(t_3a_2)]_+^{n}+[-y/(t_4a_2)]_+^{n})^{1/n}$$

and $$m(x,z)=(x^n+[z/(t_5a_3)]_+^{n}+[-z/(t_6a_3)]_+^{n})^{1/n}. \quad (18)$$

5. Applications

Once primitive soft objects are defined using field functions $(P \circ T_{s*} \circ d_i)(x,y,z)$ in Eqs. (11) and (12), soft blend, Ricci's and Perlin's set blends are allowed to have the following applications by choosing suitable modulation surfaces $f_{ci}(x,y,z)=0.5$ as described below:

(1).**Application 1**: Bulge elimination for a soft blend or a Perlin and Ricci's union blend of interconnecting or intersecting super-elliptic cylinders and super-toroids:

Even though gradient-based methods [12, 15] is not suitable for these blends to eliminate bulges due to lacking range parameters, the bulges cam also be eliminated by defining primitives by $f_i=(P \circ T_{s*} \circ d_i)(x,y,z)$, i=1,..,k, and then choosing modulation surfaces $f_{ai}(x,y,z)=0.5$ without containing the regions where unnecessary bulges lie.

As shown in Fig. 7(a), in order to eliminate bulge cylinders 1 and 2 need modulation surfaces $f_a(x,y,z)=0.5$ that do not contain the top and the bottom regions above and below the cylinders shown in Fig. 7(b). Fortunately, super-quadrics, generalized distance function, super-elliptic cylinder in Eqs. (15)-(17) offer the ability to generate or avoid a blend on the Right, Left, Top, Bottom, Front or Rear region of them, as displayed respectively in Fig. 7(c), by setting t_i, i=1-6, instead of t_ia_*, as described below:

- Setting $t_i \geq 2$, for generating a blend;
- Setting $t_i=1+\omega/a_*$, for bulge eliminations, (19)

  where $\omega \cong 0$ such as $\omega=0.1\sim0.05$ and * is the integer greater than or equal to $i/2$.

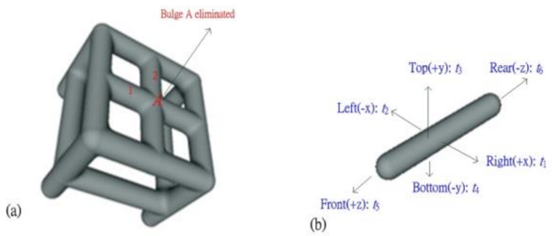

Figure 7. (a). Bulge needed to be removed. (b)-(d). The parameters for adjusting the Right, Left, Top, Bottom, Front and Rear regions of a cylinder that bulges might occur on.

For example, Fig. 8(a) shows three super-elliptic cylinders $d(x,y,z)=0.5$, $n=2$, 4, and 8:

$$d(x,y,z)=m(h(x,y), Sign(z)[|z|-16]_+)$$

where

$$h(x,y)=(|x/6|^n+|y/6|^n)^{1/n} \text{ and}$$

$$m(x,z)=(x^n+|z/6|^n)^{1/n}.$$

Fig. 8(b) shows a chair containing many bulges on the intersecting and connecting regions because it is defined by Perlin union $B_{PU}(f_1,...,f_{14})$ in Eq. (4) of cylinders $f_i=(P \circ d_i)(x,y,z)$, $i=1$ to 14 where $d_i=d(x,y,z)$ and $n=8$. However, if f_i is defined by $(P \circ T_{an} \circ d_i)(x,y,z)$ instead and modulation surface $f_{ai}=0.5$, $i=1$ to 14 is defined by Eq.(17). Thus, the bulges of Fig. 8(b) are removed as shown in Fig. 8(c) by setting $t_i=(1+0.1/6)$ to exclude those blending regions where bulges lie; for example, $f_a=0.5$ of cylinders 1 and 2 are defined by

$$f_a(x,y,z)=m(h(x,y), Sign(z)[|z|-16]_+)$$

where

$$h(x,y)=(([x/(2*6)]_+{}^n+[-x/(2*6)]_+{}^n+[y/((1+0.1/6)6)]_+{}^n+[-y/((1+0.1/6)6]]_+{}^n)^{1/n} \text{ and}$$

$$m(x,z)=(x^n+[z/((1+0.1/6)6)]_+{}^n+[-z/ ((1+0.1/6)6)]_+{}^n)^{1/n}.$$

(2). Application 2: Avoiding unwanted blends

In a union blend $B_{U3}(f_1,f_2,f_3)=0.5$, if the blending region of f_1 overlaps with those of f_2 and f_3 at two places, then two blends will be generated. However, f_1 can avoid the blend with f_3 but keep the one with f_3 unchanged, by defining them using $(P \circ T_{am} \circ d_i)$ and then choosing a modulation surface $\underline{f_{a1}}=0.5$ such that f_1's new blend region $\{(x,y,z)\in R|\ d_1(x,y,z)\geq 0.5$ and $f_{a1}(x,y,z)\leq 0.5\}$ does not touch f_3's blend region.

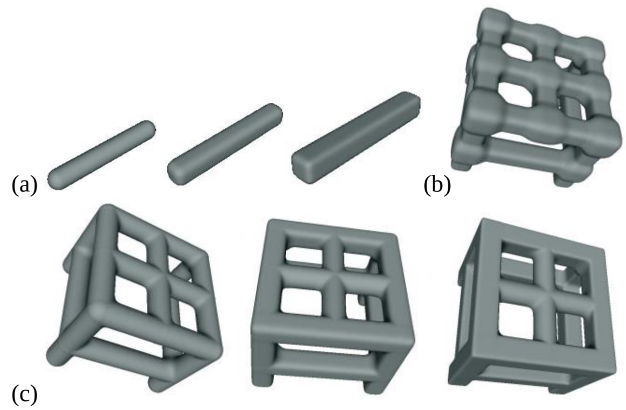

Figure 8. (a). Cylinders with contours $(x^n+y^n)^{1/n}=1$, $i=2$, 4, 8. (b). Perlin's union of interconnecting cylinders that generates bulges. (c). The bulges in (b) are eliminated by defining cylinders using $(P \circ T_{an} \circ d_i)$ and d_i and $f_{ai}(x,y,z)$ using Eq. (18), and then setting t_i by Eq. (19) to eliminate unwanted bulges.

For example, Fig. 9(d) displays a soft blend $B_{SU3}(f_1,f_2,f_3)=0.5$ of $f_i=(P_b \circ T_{an} \circ d_i)(x,y,z)=0.5$, $i=1$, 2, 3, where cylinder $f_1=0.5$ generates two blends respectively with cylinder $f_2=0.5$ and toroid $f_3=0.5$. However, one can choose a new $\underline{f_{a1}}=0.5$ for $f_1=0.5$ such that the new region $f_{a1}(x,y,z)\leq 0.5$ in Fig. 9(e) is shorter than the original one in Fig 9(f) and so the unwanted blend in region b is removed but the one in region a is still kept unchanged as in Fig. 9(g). The new and old $f_{a1}(x,y,z)$ and $d_1(x,y,z)$ are written below:

$$\text{new } f_{a1}(x,y,z)=\text{old } f_{a1}(x,y,z)=$$

$$d_1(x,y,z)=m(h(x,y), Sign(z)[|z|-16]_+)$$

where new $f_{a1}(x,y,z)$, old $f_{a1}(x,y,z)$ and $d_1(x,y,z)$ they have different definitions of $h(x,y)$ and $m(x,z)$, respectively, by

(a). $h(x,y)= (|x/6|^n+|y/6|^n)^{1/n}$ and $m(x,z)=(x^n+|z/6|^n)^{1/n}$ for d_1 ,

(b). $h(x,y)=(([x/(2*6)]_+{}^n+[-x/(2*6)]_+{}^n+[y/((1+0.1/6)6)]_+{}^n+[-y/((1+0.1/6)*6)]_+{}^n)^{1/n}$, and $m(x,z)=(x^n+[z/(2*6)]_+{}^n+[-z/(2*6)]_+{}^n)^{1/n}$ for old f_{a1}, and (c). $h(x,y)=(([x/(2*6)]_+^n+[-x/(2*6)]_+^n+[y/((1+0.1/6)6)]_+^n+[-y/((1+0.1/6)*6)]_+^n)^{1/n}$, $m(x,z)=(x^n+[z/((1+0.2/6)6)]_+^n+[-z/((1+0.2/6)6)]_+^n)^{1/n}$ for new f_{a1}.

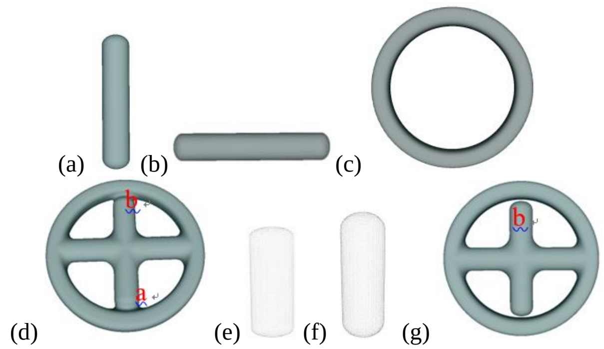

Figure 9. (a). Cylinder f_1=0.5. (b). Cylinder f_2=0.5. (c). Toroid f_3=0.5. (d). Soft blend of soft objects f_1, f_2 and defined using $f_i=(P_b \circ T_{sm} \circ d_i)$, i=1, 2, 3, on which two blends are generated at regions *a* and *b*. (e). New f_{a1}=0.5 for f_1. (f). Original f_{a1}=0.5 for f_1. (g). Because new f_{a1}=0.5 in (e) is shorter than the one in (f), the unwanted blend at region *b* is removed.

6. Conclusion

Compared to other existing blends, although soft blend, Ricci's and Perlin's set blends have lower computing complexity, they can do nothing or little for shape control, bulge elimination and avoiding unwanted blends because of lacking range parameters and the similarity of the level surfaces of existing field functions. However, if primitive soft object are defined by using the proposed field functions with surface-defined influential radii as defining functions. Because the proposed filed functions allow freely choosing modulation surfaces to change primitives' added-material and subtracted-material blend regions, soft blend, Ricci's and Perlin's set blends not only have lower computing complexity than other blends but also have the following additional applications:

(1). They can perform bulge elimination on intersecting super-elliptic cylinders and toroids although they do not have range parameters for bulge elimination using gradient-based methods.

(2). They can do bulge elimination on high-dimensional blends.

(3). They can remove unwanted blends.

Appendix

A. Definition 1: A function $f(x,y,z):R^3 \rightarrow R_+$ is called non-negative ray-linear if $f(a(x,y,z))=af(x,y,z)$ holds for any $(x,y,z) \in R^3$ and $a \in R_+$.

B. Theorem 1: If $f(x,y,z):R^3 \rightarrow R_+$ is non-negative ray-linear, then $f(x,y,z)$ can be reformulated as r/I_f, where $r=(x^2+y^2+z^2)^{0.5}$ and I_f is the distance from the origin to the intersecting point of the vector $[x,y,z]$ with the surface $f(x,y,z)=1$.

Acknowledgments

The author is thankful to IJIRTS Journal for the support to develop this document and would like to thank MOST of Taiwan for the support of Project No. MOST 109-2221-E-366-003.

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