# BACK STEPPING CONTROLLER FOR CONTAINER SHIP USING ANTI-HEELING TANKS

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#### **Abstract**

This paper presents the control of the list angle of a ship in still water based on the utilisation of anti-heeling tanks operated by pressurized air. The proposed and practical objective attempts to regulate the list angle to a prescribed value, normally zero, during the loading process of a container ship. A nonlinear adaptive controller based on the backstepping procedure has been designed as a means of counteracting the parameter inaccuracies that are involved in the nonlinear model. The main uncertainty parameters are the linear and quadratic damping coefficients, and the magnitude of the cargo that is being handled. With the objective of reducing the number of required sensors, a model reference based nonlinear observer has been introduced. The control scheme provides a robust control system under parametric uncertainties and state variables estimation errors. The designed control implies an improvement on the conventional systems.

**Keywords:** backstepping control, stability analysis, adaptive control.

#### Introduction

Two active systems based on anti-heeling water tanks, which have some sort of actuator, have been developed for controlling the heeling angle produced by an external disturbance, such as wind, waves or currents; or also due to cargo process in port of the ship, such as it is considered in this paper. In the first and oldest one developed by [1]-[4], the transfer of water between tanks (also called U-tanks) is carried out by the action of a water pump that is installed in the duct of connection between them. In the second one, [5], there is an electric motor driven force draft blower that supplies compressed air to air-valve group. The valves are controlled by an electronic control unit and they can pressurize one of the list control tanks and vent the other to create a differential pressure that would move the water between the tanks. The water difference produces the restoring torque necessary to bring the ship to an equilibrium position with a null angle of inclination. A detailed review of the development of anti-roll tanks can be found in [6]. The second method of stabilization has received relatively little attention of the researches in comparison with the first one. However, [7] developed a theory for passive U-tanks using the Euler's equation, while [8], starting from the formulation proposed by Froude [9] represented the combined motion of a ship

with a tank stabilizer by two linear differential equations, investigating the variation of the tank damping with the frequency. Subsequently, [10]-[11] proposed a simple mathematical model using an energy method and [12] used the Lagrange method to develop Kagawa's results [10]-[11] to derive the dynamic of motion for the fluid inside U-type tanks. Later on, [13] investigated the effect of tank mass and geometry on the roll angle. They concluded that under certain conditions the natural frequency of the tank can be tuned by changing the height of water in the tanks. This result was further developed by [14].

The present study deals with the design of a controller to regulate the ship's list angle in still waters by the application of the recursive procedure of the backstepping (BS) developed by [15]. The proposed control method does not need the water level measurements in the tanks or the water level difference between tanks. The only necessary measurement is the ship heel angle. The BS controller is based on an indirect adaptive control scheme where the plant is parameterized with respect to some unknown vector which is estimated by an on-line parameter estimator through processing the plant input and the output. The controller parameters are determined on basis of the estimations. The process is repeated until the performance requirements on the plant are achieved. The number of sensors has been reduced and the control shows an improvement on the existing schemes based on PID control.

The rest of this paper is organized as follows: Section 2 deals with the statement of the problem to be solved. Section 3 proposes a nonlinear model that mathematically defines the problem at hand. In Section 4 the adaptive procedure of the backstepping is designed. In Section 5, the stability of the control algorithm is analysed. In Section 6 an indirect and adaptive nonlinear control scheme is proposed and validated by simulation. Its performance is compared with the classic PI controller that can be implemented by a PLC. Finally, conclusions are drawn in Section 7.

#### **Problem Formulation**

Figure 1 shows a compressed air system with the valves' configuration that should be adopted when a list to port is produced. The center of gravity (GC) has been changed as consequence of the asymmetry of the actual load onboard. The free surface of the water in the tanks adopts the form indicated with a height H with respect to the horizontal surface. Initially,  $h_{sp}$ =0 m. Consider the case where the blowers are discharging air into

the port tank as a consequence of a list to port after the process of loading while the starboard tank is venting to the atmosphere. There is no propeller pump installed in the water cross duct between the bottoms of the tanks.

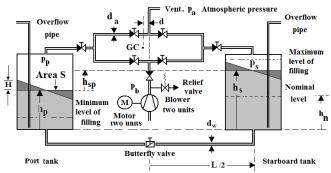


Figure 1. Scheme of the active U-tanks control system.

When the ship has an initial heel the free surface of water in the tanks is not completely horizontal. Elementary calculations show that for an angle of inclination of 1.2 degrees to port, the height H is very small (3.7 cm) (see Figure 1), dropping when the list angle decreases. From now on this gap is considered zero. The purpose of this paper is to apply the backstepping control approach in the sense of reducing the initial ship's list angle to a null value controlling the water difference between tanks  $h_{sp}$  through the pressure  $p_b$  delivered by the blowers.

The rolling motion of a ship can be described by a second – order nonlinear, ordinary differential equations, first formulated by [9], and it may be written as follows:

$$I \cdot \ddot{\phi} + M_{d}(\dot{\phi}) + M_{r}(\phi) + M_{ab}(h) = M_{I}(t) \tag{1}$$

where I is the virtual moment of inertia of the ship with respect to transversal inclinations, i.e., the sum of the moment of inertia of the real mass of the ship and the contribution from the hydrodynamical effect of the added mass,  $\phi$  is the heeling angle with respect to the calm sea surface (the dot indicates derivation with respect to time),  $M_{ah}$  the torque produced by the anti-heeling system to counteract the effect produced by the asymmetric load distribution,  $M_d(\dot{\phi})$  is the moment of the dissipative forces generated during the oscillatory motion,  $M_r(\phi)$  is the righting moment and  $M_L(t)$  is the external moment produced by an asymmetry in the load distribution on the ship. The moment of inertia is calculated on through the equation that defines the ship's free rolling period,

$$T = 2 \cdot \pi \cdot \sqrt{\frac{I}{m \cdot g \cdot GM}} \tag{2}$$

where m is the ship mass, g is the acceleration due to gravity and GM the still water transverse metacentric height. The functional relation between the moment due to dissipative forces and the roll rate  $M_d(\dot{\phi})$  has been investigated by [16-17] and [18]. These investigators have formulated a linear plus-quadratic damping dependency

$$M_d(\dot{\phi}) = d_1 \cdot \dot{\phi} + d_2 \cdot \dot{\phi} \cdot |\dot{\phi}| \tag{3}$$

where the constants  $d_1$  and  $d_2$  were estimated by [19] for several types of ships. At the same time, if  $\Delta$  represents the ship's displacement, the restoring moment  $M_r(\phi)$  is modeled as

$$M_{x}(\phi) = \Delta \cdot GZ(\phi) = \Delta \cdot (c_1 \cdot \phi + c_3 \cdot \phi^3 + c_5 \cdot \phi^5 + c_7 \cdot \phi^7) \quad (4)$$

The experimental values of the righting arm GZ are plotted against the heel angle  $\phi$ . The fitting for the considered ship was obtained by least squares method and it is shown in Figure 2 while in Table 1 the values of the coefficients are reported. Because of the asymmetry in load distribution the actual center of gravity is located to 0.03 m from the ship's centerline towards port and 7.30 m from amidships perpendicular to bow.

The moment produced by the anti-heeling system can be modeled as,

$$M_{ah} = \frac{S \cdot \gamma_w \cdot h_{sp} \cdot L}{2} \tag{5}$$

where S is the free surface of the tank that will be utilized for correcting the ship list and  $h_{sp}$  the water level difference between tanks,  $\gamma_w$  the fresh water specific gravity, while L represents the arm of the torque produced by the anti-heeling tanks (see Fig.1). Introducing the new variables  $x_1 = \phi$  and  $x_2 = \dot{\phi}$ , and after substituting (3), (4) and (5) into (1) the dynamics of the system can be rewritten as

$$\dot{x}_{1} = x_{2} \qquad (6a)$$

$$\dot{x}_{2} = -\left[\alpha_{I} + \alpha_{2} \cdot |x_{2}|\right] \cdot x_{2} - \sum_{j=1}^{4} \beta_{2:j-1} \cdot x_{1}^{2:j-1} - \delta \cdot h_{sp} + M_{LN} \qquad (6b)$$

$$y = x_{1} \qquad (6c)$$

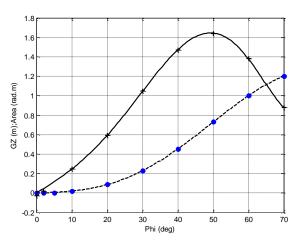


Figure 2. Static (—) and dynamic stability (-----) of the fast ferry ship under study during the load process.  $(+, \bullet)$  Experimental points. The lines represent the fittings obtained by the least squares method

Table 1. Parameters of the water/air circuits

Air pipe diamete	r			d <sub>a</sub> =300	
				mm	
Water pipe diame	eter			$d_{w} = 50$	
				0  mm	
Stabilizer tanks (length, width, height) (m)					
Tank A	Tank B		Tank C		
7.38, 3.50, 3.30	7.38, 3.50, 3.34	7.38	, 3.50, 7.94		
Tanks: Nominal level of filling h <sub>n</sub> =3.60 m					
Number of blowe	er units			2	
Differential pressure (max) (one blower)			800		
				mbar	
Maximum anti-heeling moment			3638		
				tons.m	
Distance L (see I	Figure 1)			11.5 m	
Coefficients of Eq.(4)					
$c_1=1.299 \text{ (m/rad)}$ $c_3=3.873 \text{ (m/rad}^3)$ $c_5=-5.355$					
$(m/rad^5)$ $c_7=$	$=1.676 \text{ (m/rad}^7)$				

The variable  $h_{sp}$  represents the intermediate control that should be supplied by definitive control, the pressure blower  $p_{b_i}$  while y is the output to be controlled by the nonlinear controller. The parameters  $\alpha_i$ ,  $\beta_{2:i-1}$ ,  $\delta$ , have been defined as,

$$\alpha_{i} = \frac{d_{i}}{I} (i = 1, 2) \quad (7a)$$

$$\beta_{2 \cdot j - l} = \frac{\Delta \cdot c_{2 \cdot j - l}}{I} (j = 1 \cdot \cdot \cdot 4) \quad (7b)$$

$$\delta = \frac{S \cdot \gamma_{w} \cdot L}{2 \cdot l} \quad (7c)$$

$$M_{LN} = \frac{M_L}{I} \tag{7d}$$

In the procedure carried out, three considerations have been taken into account:

- 1) With the purpose of avoiding the differentiation of the heel angle or using some derivative filters that involve the measures of the list sensor to obtain the velocity of the roll, a reduced order closed-loop observer is introduced.
- 2) At the same time, an initial estimation of the roll damping is extremely ambiguous owing the highly nonlinear nature of the motion. Although extensive theoretical research developed by [20] and [21], experimental works and numerical simulations studies have been conducted on the matter, it is still far from being complete. The controller that will be designed can cope with uncertainties on these parameters once a functional expression of damping model has been proposed. In this sense, the coefficients  $\alpha_i(i=1,2)$  must be replaced in (6b) and (7a) by their estimates  $\hat{\alpha}_i(i=1,2)$  due to the uncertainties in the damping constants  $d_i(i=1,2)$ . The same considerations have to be taken with  $x_2$  in (6a) and (6b), and  $M_{LN}$  in (7d) as consequence of the size of the load that being getting onboard and the actual ship's inertial moment.
- 3)The control objective should be achieved when the control system does not have information about the loading produced at the initial heel angle.

The purpose of the control system is to control the list angle, being used a state observer to determine the angular velocity. In this sense, the observer [22] is defined as,

$$\dot{\hat{x}}_2 = \varepsilon \left( x_I, \hat{x}_2, \hat{\alpha}_i, h_{sp}, \hat{M}_{LN} \right) + K \cdot \tilde{x}_2 \tag{8}$$

where the gain K is a design parameter. Its usefulness will be tested later when the stability of the whole systems is analyzed. The nonlinear function  $\varepsilon(\cdot)$  is given by

$$\varepsilon(x_{1}, \hat{x}_{2}, \hat{\alpha}_{i}, h_{sp}, \hat{M}_{LN}) = -H(|\hat{x}_{2}|, \hat{\alpha}_{1}, \hat{\alpha}_{2}) \cdot \hat{x}_{2} + \\ -\sum_{j=1}^{4} \beta_{2 \cdot j-1} \cdot x_{1}^{2 \cdot j-1} - \delta \cdot h_{sp} + \hat{M}_{LN}$$
(9)

where  $\hat{x}_2$ ,  $\hat{M}_{LN}$ ,  $\hat{\alpha}_i$  (i=1,2) are the estimates of  $x_2$ ,  $M_{LN}$  and  $\alpha_i$  (i=1,2), respectively,  $\tilde{x}_2=x_2-\hat{x}_2$ ,  $\tilde{M}_{LN}=M_{LN}-\hat{M}_{LN}$ ,  $\tilde{\alpha}_i=\alpha_i-\hat{\alpha}_i$  (i=1,2) are the observation and estimation errors

and 
$$H(|\hat{x}_2|, \hat{\alpha}_1, \hat{\alpha}_2) = \sum_{i=1}^{2} |\hat{\alpha}_i| \cdot |\hat{x}_2|^{i-1} = \hat{\alpha}_1 + \hat{\alpha}_2 \cdot |\hat{x}_2|$$
, is the

estimate of the linear function

$$H(|x_2|, \alpha_1, \alpha_2) = \alpha_1 + \alpha_2 \cdot |x_2| \tag{10}$$

#### **Backstepping Controller Design**

The relative order of the considered system is two, therefore the backstepping process should be developed in two steps.

Step1. Introducing the variable  $z_1$  representing the tracking error,

$$z_1(t) = x_1(t) - x_{1d}(t) \tag{11}$$

where  $x_{Id}$  (t) represents the desired behavior of the output  $x_{I} = \phi$ , the ship's list. Its temporal variation is given by,

$$\dot{z}_1 = \dot{x}_1 - \dot{x}_{1d} = x_2 - \dot{x}_{1d} = \tilde{x}_2 + \hat{x}_2 - \dot{x}_{1d} \quad (12)$$

The main idea of the backstepping is to choose one of the state space variables as virtual control input. As consequence of the use of an observer to estimate the state  $x_2$ , it is possible to choose the virtual control as the sum of an error variable  $z_2$  (defined in the second step of the backstepping), with a stabilizing function  $\Xi$ ,

$$\hat{x}_2 = z_2 + \Xi \tag{13}$$

By inserting (13) into (12) the temporal variation of the first error variable  $z_I$  reads

$$\dot{z}_1 = \tilde{x}_2 + z_2 + \Xi - \dot{x}_{1d} \qquad (14)$$

The stabilizing function is then chosen as,

$$\Xi = -C_1 \cdot z_1 - D_1 \cdot z_1 + \dot{x}_{Id} \tag{15}$$

being  $C_1$  and  $D_1$  two elements of the matrices  $C = diag(C_1 \quad C_2)$ ,  $D = diag(D_1 \quad D_2)$  both diagonals and positives, named, feedback and damping matrices, respectively. It has been necessary to add the damping term, as consequence of the term  $\widetilde{X}_2$  in (14), which can be considered as a perturbation in the  $z_1$  dynamics, so that its influence should be compensated. After substituting (15) in (14), it leads to

$$\dot{z}_1 = -(C_1 + D_1) \cdot z_1 + z_2 + \tilde{x}_2 \quad (16)$$

Step 2. In the second step of the backstepping it is necessary to consider the dynamics of the second error variable  $z_2$ . Taking the derivatives of  $z_2$  in (13) and of  $\Xi$  from (15) the following is obtained

$$\dot{z}_{2} = \dot{\hat{x}}_{2} + (C_{1} + D_{1}) \cdot \dot{z}_{1} - \ddot{x}_{1d} = \dot{\hat{x}}_{2} - (C_{1} + D_{1})^{2} \cdot z_{1} + (C_{1} + D_{1}) \cdot (z_{2} + \widetilde{x}_{2}) - \ddot{x}_{1d}$$

$$(17)$$

As consequence that the variation of the state  $x_2(t)$  is estimated by (8) and after considering (9), it is possible to write (17) as,

$$\dot{z}_{2} = -\delta \cdot h_{sp} - H(|\hat{x}_{2}|, \hat{\alpha}_{1}, \hat{\alpha}_{2}) \cdot \hat{x}_{2} - \sum_{j=1}^{4} \beta_{2 \cdot j-1} \cdot x_{1}^{2 \cdot j-1} + 
+ \hat{M}_{LN} + K \cdot \tilde{x}_{2} - (C_{1} + D_{1})^{2} \cdot z_{1} + 
+ (C_{1} + D_{1}) \cdot (z_{2} + \tilde{x}_{2}) - \ddot{x}_{1d}$$
(18)

One of the main characteristics of the backstepping procedure is its ability to tolerate the selection of an intermediate control  $h_{sp}$  according to the design convenience so that the second state  $z_2$  depends on the parameters  $C_2$  and  $D_2$  of the feedback and damping matrices which could be selected in an arbitrary fashion to guarantee the system stability. The intermediate control  $h_{sp}$  is chosen by convenience as,

$$h_{sp} = \frac{-(C_1 + D_1)^2 \cdot z_1 + (C_1 + D_1 + C_2 + D_2) \cdot z_2 - \ddot{x}_{1d} + z_1 + \mu_1}{\delta}$$
(19)

where  $\mu_1$  is given by,

$$\mu_{1} = -H(\hat{x}_{2}|, \hat{\alpha}_{1}, \hat{\alpha}_{2}) \cdot \hat{x}_{2} - \sum_{j=1}^{4} \beta_{2 \cdot j-1} \cdot x_{1}^{2 \cdot j-1} + \hat{M}_{LN}$$

Thus, by means of (18) and (19), the temporal variation of the second error variable is given by,

$$\dot{z}_2 = -z_1 - (C_2 + D_2) \cdot z_2 + \Omega \cdot \widetilde{x}_2 \tag{20a}$$

$$\Omega = (C_I + D_I) + K \tag{20b}$$

In matrix form (16) and (20a), yields

$$\dot{z} = -(C + D + E) \cdot z + W \cdot \widetilde{x}_2 \qquad (21)$$

where 
$$E = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$$
;  $z = \begin{pmatrix} z_1 & z_2 \end{pmatrix}^T$ ;  $W = \begin{pmatrix} 1 & \Omega \end{pmatrix}^T$ 

Equations (16) and (20a) cannot be implemented because the error on the heel rate  $\tilde{x}_2$  is unknown. To overcome this difficulty, it is necessary to determine the error that has been achieved by the observer. Starting by subtracting (6b) and (8), the following equality is obtained,

$$\dot{\mathbf{x}}_{2} - \dot{\hat{\mathbf{x}}}_{2} = \dot{\hat{\mathbf{x}}}_{2} = -H(|\mathbf{x}_{2}|, \alpha_{1}, \alpha_{2}) \cdot \mathbf{x}_{2} - \sum_{j=1}^{4} \beta_{2 \cdot j-1} \cdot x_{1}^{2 \cdot j-1} + \\ -\delta \cdot h_{sp} + M_{LN} - \varepsilon(\mathbf{x}_{1}, \hat{\mathbf{x}}_{2}, \hat{\alpha}_{i}, h, \hat{M}_{LN}) \cdot \mathbf{K} \cdot \tilde{\mathbf{x}}_{2}$$
(22)

After substituting in the former equation the term  $\delta \cdot h_{sp}$  given by the expression (19), adding and subtracting the same quantity  $H(|\hat{x}_2|, \alpha_1, \alpha_2) \cdot \hat{x}_2$ , it is easy to verify that

$$\begin{split} \dot{\tilde{x}}_2 &= -H(|\hat{x}_2|, \alpha_1, \alpha_2) \cdot x_2 + H(|\hat{x}_2|, \alpha_1, \alpha_2) \cdot \hat{x}_2 - K \cdot \tilde{x}_2 - \Theta \quad \text{(23a)} \\ \Theta &= \left[ H(|\hat{x}_2|, \alpha_1, \alpha_2) \cdot \hat{x}_2 - M_{LN} \right] - \left[ H(|\hat{x}_2|, \hat{\alpha}_1, \hat{\alpha}_2) \cdot \hat{x}_2 - \hat{M}_{LN} \right] \text{(23b)} \end{split}$$
 The function  $\Theta$  is related to the vectors  $\sigma = \begin{pmatrix} \alpha_1 & \alpha_2 & M_{LN} \end{pmatrix}^T$  and  $\hat{\sigma} = \begin{pmatrix} \hat{\alpha}_1 & \hat{\alpha}_1 & \hat{M}_{LN} \end{pmatrix}^T$  that have been defined with the purpose of grouping the unknown

parameters. After defining  $\varphi(x_2) = (x_2 | x_2 | \cdot x_2 - I)$  and  $\varphi(\hat{x}_2) = (\hat{x}_2 | \hat{x}_2 | \cdot \hat{x}_2 - I)$ , if  $\hat{x}_2$  matches with its true value, it results.

$$\Theta = [\varphi(\hat{x}_2) \cdot \sigma] - [\varphi(\hat{x}_2) \cdot \hat{\sigma}] = \varphi(\hat{x}_2) \cdot (\sigma - \hat{\sigma}) = \varphi(\hat{x}_2) \cdot \tilde{\sigma}$$
(24)

## Control System Stability

It is necessary to verify that the control law, together with the updating parameter laws that have been previously designed, provide the robust closed loop stability and consequently the objectives of the tracking control problem. In order to verify the system stability the following candidate Lyapunov function is introduced,

$$V(z, \widetilde{x}_2, \widetilde{\sigma}) = \frac{1}{2} \cdot \left[ z^T \cdot z + \widetilde{x}_2^2 + \widetilde{\sigma}^T \cdot \Gamma^{-1} \cdot \widetilde{\sigma} \right]$$
 (25)

where  $\Gamma$  is a diagonal matrix of non dimensional gains  $\Gamma_{I}, \Gamma_{2}, \Gamma_{IN}$ . The temporal derivative of V is,

$$\dot{V}(z, \widetilde{x}_2, \widetilde{\sigma}) = z^T \cdot \dot{z} + \dot{\widetilde{x}}_2 \cdot \widetilde{x}_2 + \widetilde{\sigma}^T \cdot \Gamma^{-l} \cdot \dot{\widetilde{\sigma}}$$
 (26)

Replacing in (26) the expression of  $\dot{z}$  given by (21) and  $\dot{\tilde{x}}_2$  obtained from (23a), after adding and subtracting  $\tilde{x}_2 \cdot H(|x_2|, \alpha_1, \alpha_2) \cdot \tilde{x}_2$ , with the special feature of verifying that  $z^T \cdot E \cdot z = 0$ , yields,

$$\dot{\mathbf{V}}(\mathbf{z}, \widetilde{\mathbf{x}}_{2}, \widetilde{\boldsymbol{\sigma}}) = -\mathbf{z}^{\mathsf{T}} \cdot (\mathbf{C} + \mathbf{D}) \cdot \mathbf{z} + + \mathbf{z}^{\mathsf{T}} \cdot \mathbf{W} \cdot \widetilde{\mathbf{x}}_{2} - \widetilde{\mathbf{x}}_{2}^{2} \cdot [H(|\widehat{\mathbf{x}}_{2}|, \alpha_{1}, \alpha_{2}) + \mathbf{K}] + - \widetilde{\mathbf{x}}_{2} \cdot \cdot \widetilde{\boldsymbol{\sigma}} + \widetilde{\boldsymbol{\sigma}}^{\mathsf{T}} \cdot \Gamma^{-1} \cdot \dot{\widetilde{\boldsymbol{\sigma}}}$$

$$(27)$$

In order to achieve the required robust stability with respect to the error in the vector of unknown but constant parameters  $(\dot{\tilde{\sigma}} = -\dot{\tilde{\sigma}})$  and in the measures of the state  $x_2$  carried out during the estimation, it is necessary that in the second member of (27), the last two adding elements become zero, which is achieved by applying the following estimation law

$$\dot{\hat{\sigma}} = -\Gamma \cdot \varphi^T \cdot \tilde{x}_2 = -\begin{pmatrix} \Gamma_I & 0 & 0 \\ 0 & \Gamma_2 & 0 \\ 0 & 0 & \Gamma_{LN} \end{pmatrix} \cdot \varphi^T \cdot \tilde{x}_2 \tag{28}$$

Inserting (28) into  $\dot{V}$ , the (27) becomes,

$$\dot{\mathbf{V}}(\mathbf{z}, \widetilde{\mathbf{x}}_{2}, \widetilde{\boldsymbol{\sigma}}) = -\mathbf{z}^{\mathrm{T}} \cdot (\mathbf{C} + \mathbf{D}) \cdot \mathbf{z} + \mathbf{z}^{\mathrm{T}} \cdot \mathbf{W} \cdot \widetilde{\mathbf{x}}_{2} + \\ -\widetilde{\mathbf{x}}_{2}^{2} \cdot \left[ H(|\widehat{\mathbf{x}}_{2}|, \alpha_{1}, \alpha_{2}) + \mathbf{K} \right]$$
(29)

The function V should be semi-definite negative. Observing the last term of the second member in (29), it is necessary that the conditions a) and b) are to be satisfied:

a) 
$$H(|\hat{x}_{2}|, \hat{\alpha}_{1}, \hat{\alpha}_{2}) + K \ge 0$$

From the definition of  $H(|\hat{x}_2|, \alpha_1, \alpha_2)$  given by (10), this condition implies that

$$K \ge -\left[\hat{\alpha}_1 + \hat{\alpha}_2 \cdot \left| \hat{x}_2 \right| \right] \tag{30}$$

The gain *K* must be positive as consequence of that the estimates are strictly positives with the same property that the elements of the damping matrix.

b) 
$$\left[ -z^T \cdot (C+D) \cdot z + z^T \cdot W \cdot \widetilde{x}_2 \right] \le 0$$

This condition is satisfied after considering that

$$-z^{T} \cdot (C+D) \cdot z + \tilde{z}^{T} \cdot W \cdot \tilde{x}_{2} = -[(C_{1}+D_{1}) \cdot z_{1}^{2} - z_{1} \cdot \tilde{x}_{2}] + -[(C_{2}+D_{2}) \cdot z_{2}^{2} - z_{2} \cdot (C_{1}+D_{1}+K) \cdot \tilde{x}_{2}]$$

Adding and subtracting the quantities  $\tilde{\chi}_2^2/[4(C_1+D_1)]$  and  $(C_1+D_1+K)\cdot\tilde{\chi}_2^2/[4(C_2+D_2)]$ , the following identity is obtained,

$$-\left[z^{T} \cdot (C+D) \cdot z - z^{T} \cdot W \cdot \widetilde{x}_{2}\right] = -\left(A^{2} + B^{2}\right) +$$

$$-\frac{1}{4} \cdot \left[\frac{1}{C_{1} + D_{1}} + \frac{C_{1} + D_{1} + K}{C_{2} + D_{2}}\right] \cdot x_{2}^{2} \le 0$$
(31)

Where A and B are respectively defined as follows:

$$A = \left[ \frac{2 \cdot (C_1 + D_1) \cdot z_1 - \tilde{x}_2}{2 \cdot (C_1 + D_1)^{1/2}} \right]$$
 (32a)

$$B = \left[ \frac{2 \cdot (C_2 + D_2) \cdot z_2 - (C_1 + D_1 + K) \cdot \widetilde{x}_2}{2 \cdot (C_2 + D_2)^{1/2}} \right]$$
(32b)

As consequence of the positive values of the coefficients  $(C_i,D_i, i=1,2)$  and K, the condition b) is always fulfilled. Whenever condition a) is fulfilled, the following inequality holds:

$$\dot{V}(z, \tilde{x}_2, \tilde{\sigma}) \le 0$$
 (33)

Since V is a continuously differentiable function, positive definite and radially unbounded and  $\dot{V} \leq 0$ , it follows from the Krasovskii- LaSalle theorem [15], that in the  $(z_I,z_2)$  coordinates, the equilibrium state (0,0) is globally asymptotically stable and the estimations of the parameters  $\alpha_i(t), i=1,2$  and  $M_{LN}$  are globally bounded, due to the positive values of the adaptive gains,  $\Gamma_I, \Gamma_2, \Gamma_{LN}$ . Based on (11),  $z_I$  go to zero in an asymptotic manner and  $x_I(t) \rightarrow x_{Id}(t)$  when  $t \rightarrow \infty$ . The asymptotic stability of  $z_2$ , jointly with  $z_I$ , the bound on the second derivative of the desired trajectory and after considering the last terms of (16), the observer errors tend to a null value when the time is increased.

## Simulation Analysis

There are two types of blowers commonly used in aeration applications, namely the positive displacement blowers (PDB) and the centrifugal blowers (CB). The (PDB) provides practically constant flow at widely varying discharge pressures. Low capital costs and its ability to operate at widely varying pressures are the advantages of this blower. The capacity control of a (PDB) can be achieved either by blowing off excess air through a blow-off valve or by varying the speed of the blower using a variable-frequency drive. The first method of blowing off the excess air during low air demand period wastes energy and is not recommended. Reducing air flow by reducing the blower speed is a better way of controlling air flow, because it conserves energy by reducing the horsepower requirement. The characteristic water level difference  $h_{sp}(m)$  versus pressure  $p_b(mb)$  of a typical blower is shown in Equation (34).

$$p_b = 1018 + 98.17 \cdot h_{sp} \tag{34}$$

In order to check the proposed control perfomance, the list angle response under an input disturbance (load change) is shown in Figure 3. The disturbance load consists in a truck of 50 tons, 12,19 m long (40 ft) loaded by port side with a speed of 1.39 m/s (5 km/h).

The correction rate that should be supplied by the antiheeling system is about 59.66 tons.m/s, until reaching the full load in 8.8 s. From this instant the load is kept, and it delivers a constant torque of 525 tons.m. (see the left upper block in Figure 5). The load is carried out at a distance of 10.50 m of the centerline of the ship whose characteristics are shown in Table 2. The different torques involved in the proposed case are shown in Figure 4. For the ship's stabilization purpose it is only necessary the utilization of tank A (see Table 1), whose maximum torque is 823.52 tons.m (One tank in its minimum level 0.5 m and the opposite in its maximum level 6.70 m). The designed control system tries to minimize the heeling angle.

The simulated case study provides us a clear insight high-lighting the differences between the classic PI controller whose control scheme is depicted in Figure 5 and the proposed controller based on the backstepping procedure (Figure 7). For the constants given in Table 3, the time evolution of the different variables are shown in Figure 6. The response has an overshoot of 0.96 deg at time of 22 s, reaching the desired heel angle in 90 s.

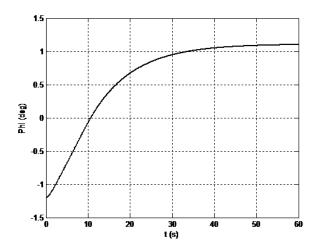


Figure 3. Temporal variation of the heel angle when a load of 50 tons at a speed of 5 km/h is getting on board. Its final value is 1.1 deg and the initial one -1.2 degrees

Table 2. Relevant characteristics of the ship in the analyzed control problem

Displacement	$\Delta$ =20852	Light	10153.6
	2 tons	weight	tons
Metacentric	GM=1.54	Dead	10698.6
height	m	weight	tons
Length be-	$L_{pp} = 170$	Center of	gravity of
tween perpen-	m	the ship under load-	
diculars		ing conditions	
Draught at	7.412 m	-7.30 m	Towards
midship			bow
Rolling period	15.3 s	d=-0.03	Towards
		m	port
Block coeffi-			
cient	0.647		
Inertia mo-	$1.90.10^5$	Breadth	25.20 m
ment	tons.m <sup>2</sup>	(moulded)	

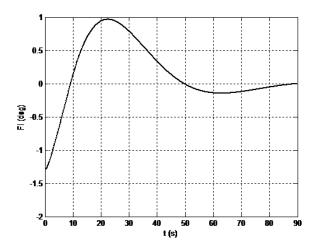


Figure 6a. Temporal variation of the list angle with a PI controller

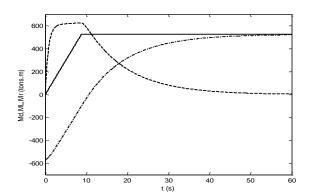


Fig. 4. Temporal variation of the damping moment  $M_d$  (dashed line), external moment  $M_L$  (solid line) and the righting moment  $M_r$  (dashdot line)

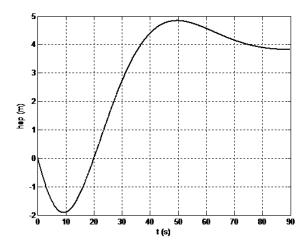


Figure 6b. Temporal variation of the gap between the starboard and port tanks when a PI controller is used

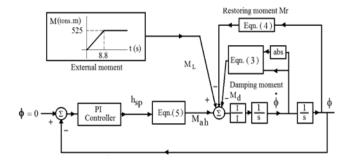


Figure .5. Control structure with a conventional PI controller

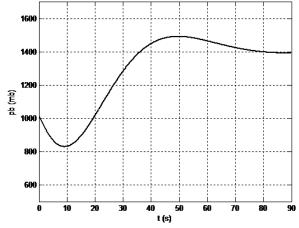


Figure 6c. Temporal variation of the pressure delivered by the blowers when a PI controller is used

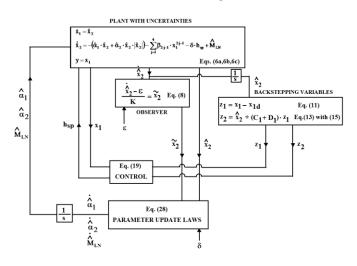


Figure 7. Scheme of the adaptive backstepping control system proposed in this paper

Table 3. Controllers parameters used in simulations

PI controller (p.u)	Backstepping controller (p.u)		
Proportional gain	$C_1+D_1$	0.1	
0.579	$C_2 + D_2$	1	
Integral gain	$\Gamma_{1}$	749428	
17	$\Gamma_{2}$	$10^{7}$	
1 /	$\Gamma_{ m LN}$	99.6	

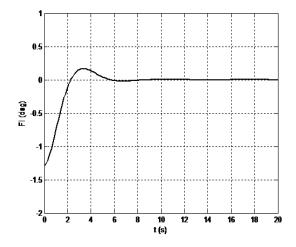


Figure 8a. Temporal variation of the list angle with a backstepping controller

The tuning procedure requires a preliminary test by handling some cargo on the ship where the list angle is recorded as well as the step like pressure supplied by the blowers. On the contrary, the procedure based on the backstepping algorithm, whose scheme is depicted Figure 7, does not require any test as in the last case admitting uncertainties on the parameters that are difficult to know in an exact manner such as the damping coefficients and the weight that are being loaded. The simulation results are shown in Figures 8 (a, b, c, d, e).

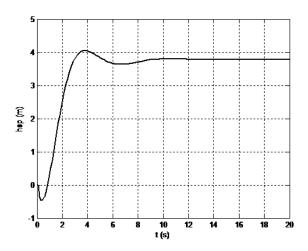


Figure 8b. Temporal variation of the gap between the starboard and port tanks when a backstepping controller is used

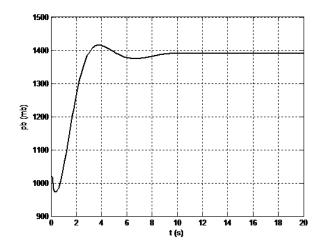


Figure 8c. Temporal variation of the pressure delivered by the blowers when a backstepping controller is used

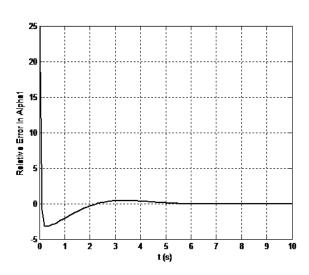


Figure 8d. Temporal variation of the relative error in parameter  $\alpha_I$  when a backstepping controller is used

The backstepping controller response shows a shorter time for reaching the control objective, a smaller overshoot, and shorter peak time and settling time. Moreover, it is not necessary a previous tuning process for the computation of the controller parameters.

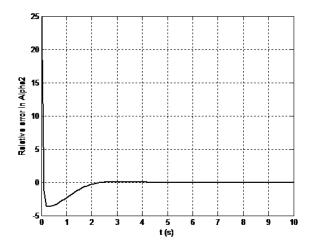


Figure 8e. Temporal variation of the relative error in parameter  $\alpha_2$  when a backstepping controller is used

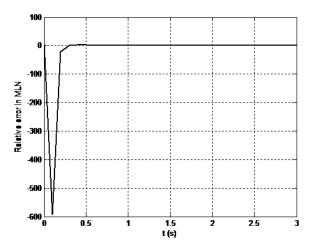


Figure 8f. Temporal variation of the relative error in parameter  $\,M_{\rm NL}$  when a backstepping controller is used

#### Conclusions

The procedure of applying the backstepping algorithm to control the heeling angle of a ship in still waters has been presented. The proposed control system is capable of carrying out the control of the inclination angle of a ship when an unknown amount of cargo is handled not only at any cargo space but also at any ship place, including structural weights restricted only to the ship's static stability. Results show a dynamic response exempt of overshooting for any cargo operation.

With the purpose of reducing the number of sensors a reduced order observer and a state estimator according to an indirect scheme of adaptation on the basis of a reference model design technique has been introduced. Its robustness has been proved, showing a stable behaviour.

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