ISSN:2321-1156
International Journal of Innovative Research in Technology & Science(IJIRTS)

Some Results on Weak Multiplication Modules

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Abstract: In this paper some results on weak multiplication modules have been given.

Keywords: Prime Submodules, multiplication module, weak multiplication module.**MSC 2000:** 13E05

1. Introduction:

Multiplication modules was introduced by Barnard [1] in 1981 and after some time using the concept prime submodules of modules, the concept of weak multiplication modules was developed and many more results have been given [5]. In this line of research Nezhad and Naderi [6] has also defined the notion of residual of a submodule in a module and obtained some related results on prime and semiprime submodules of multiplication modules. This paper continues this line of research for weak multiplication modules.

Throughout this paper all rings are commutative with identity and all modules are unitary. If N and K are submodules of R-module M then the residual ideal N by K is defined as $(N :_R K) = \{ r \in R : rK \subseteq N\}$. Let N be submodule of M and I be an ideal of R the residual submodule N by I is defined as $(M :_M I) = \{ m \in M : mI \subseteq N\}$.

In the special case in which N= 0 the ideal $(0 :_R K)$ is called annihilator of K and it is denoted by $Ann_R(K)$ also the submodule $(0 :_M I)$ is called the annihilator of in M and it is denoted by $Ann_M(I)$.

2. Preliminaries:

In this section we give some basic definitions, theorems and propositions related to weak multiplication modules which are useful to understand the further results:

Definition 2.1 [1] An R-module M is called a multiplication module if every submodule N of M, we have N = IM, where I is an ideal of R.

Definition 2.2 [7] A proper submodule N of an R-module M is said to be prime submodule of M if $ra \in N$ for $r \in R$ and $a \in M$ then either $a \in N$ or $rM \subseteq N$ (also see examples in [3], [4]).

Here it is remarkable that N is prime submodule of M then $P = (N :_R M)$ is necessarily a prime ideal of R and therefore N is sometime referred as P-prime submodule of M [7].

Definition 2.3 [5] An R-module M is called weak multiplication module if M doesn't have any prime submodule or every prime submodule N of M, we have N = IM , where I is an ideal of R.

One can easily show that if an R-module M is a weak multiplication module then $N = (N :_R M)M$ for every prime submodule N of M [2].

Theorem 2.4 [6] Let M be a multiplication R-module and N a proper submodule of M. Then following statements are equivalent:

1. N is a prime submodule of M.
2. $Ann_R(M/N)$ is a prime ideal of R.
3. N = PM for some prime ideal P of R with $Ann_R M \subseteq P$.

Theorem 2.5 [6] Let P be a proper submodule of multiplication R-module M. Then the following statements are equivalent:

1. P is prime submodule.
2. For every submodule N and K of M if $NK \subseteq P$ then either $N \subseteq P$ or $K \subseteq P$;
3. For every m, n $\in M$, if $mn \subseteq P$ then either $m \in P$ or $n \in P$.

Definition 2.6 [6] Let M be a multiplication R-module and N and L be submodules of M. The residual of L by N in M is
$(L :_M N) = \{ m \in M : mn \subseteq L$ for every $n \in N \}$

We will call $(0 :_M N)$ annihilator of N in M and denoted by $Ann_M(N)$.

Proposition 2.7 [6] Let M be a multiplication R-module and L and N be two submodules of M. Then $(L :_M N) = (L :_R N) M$.
In particular $Ann_M(N) = (Ann_R(N)) M$.

Proposition 2.8 [6] Let M be a multiplication R-module and L be a proper submodule of M . Then the following statements are equivalent:

1. L is a prime submodule of M.
2. For every submodule N of M, if $N \nsubseteq L$ then $(L :_M N) = L$.

Theorem 2.9 [8] Every non zero multiplication module has maximal submodules.

3. Main Results:

In this section we investigate the following results:

Proposition 3.1 Let M be a multiplication R-module and N and L be submodules of M such that $L \nsubseteq N$. If L is a prime submodule of N then $(L :_M N)$ is a prime submodule of M and so M is weak multiplication module.

Proof: If L is a prime submodule of N then by remark $(L :_R N)$ is a prime ideal of R and $Ann_R(M) \subseteq (L :_R N)$ and so $(L :_R M)M$ is a prime submodule of M(by theorem 2.4). Therefore $(L :_M N)$ is a prime submodule of M (by proposition 2.7) and so M is weak multiplication module.

Proposition 3.2 Let M be a multiplication R-module, N a maximal submodule of M and P a prime submodule of M such that $P \neq N$. If L is a submodule of P then $(L :_M N)$ is also a submodule of P. In Particular, if $(L :_M N)$ is a prime submodule of M then $(L :_M N)$ is a minimal prime submodule of L and so M is weak multiplication module.

Proof: Let L be submodule of P and $m \in (L :_M N)$. Hence $mN \subseteq L \subseteq P$. If $m \notin P$ then $N \subseteq P$ by theorem 2.5 and so $N = P$, because N is a maximal submodule of M , which is a contradiction. Therefore $m \in P$. Therefore $(L :_M N) \subseteq P$. In particular, if $(L :_M N)$ is a prime submodule of M then $(L :_M N)$ is a minimal prime submodule of L and so M is weak multiplication module.

Corollary 3.3 Let M be a multiplication R-module and N be a maximal submodule of M. If $Ann_M(N)$ is a prime submodule of M then $Ann_M(N)$ is a minimal prime submodule of M and so M is weak multiplication module.

Acknowledgment:
The author would like to thanks CCOST RAIPUR (Chhattisgarh Council of Science

and Technology Raipur C. G. INDIA) for the financial support.

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