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VARIABLE INFLUENTIAL RADIUS METHOD FOR SHAPE CONTROL ON THE TRANSITION OF UNION BLENDING SURFACES IN SOFT OBJECT MODELING

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Abstract

In computer graphics or CAD, soft object modeling defines a soft object as a level surface of a defining function called field function. Furthermore, it provides an easy way to define a more complex soft object by connecting primitive soft objects smoothly with automatically generated transition via blending operations, such as union and intersection. In the literature, a lot of blending operations were developed and these include blending operations with or without blending range control and soft blend. However, existing blending operations still face a difficulty that the shape of the contour of the automatically generated transition of a union blend is always similar to that of blended soft objects. For example, if a cylinder is blended with a cube, then the contour of the transition will be a circle. To conquer this problem, this paper proposes variable influential radius method. This method allows one to freely choose an implicit contour surface. And then the contour surface is used to vary the influential radius of the field functions of blended soft objects and hence the shape of the contour of the transition of the resulting blending surface can be similar to that of the chosen contour surface. That is, this method enables that one can freely choose a contour surface to determine and control the shape of the transition of a union blending surface in soft object modeling.

I. Introduction

In computer graphics or computer-aided design, object modeling studies how to represent the shape of objects. Among existing object modeling techniques, implicit surface modeling is attracting attention because it provides an easy way to create a complex implicit surface by connecting primitive surfaces, such as planes and super-ellipsoids by Boolean set operations (union, intersection and difference).

In implicit surface modeling, an implicit surface is defined as a level surface of a defining function $fi(x,y,z)$ by $fi(x,y,z)=C$. In fact, $fi(x,y,z)$ determines the shape of $fi(x,y,z)=C$. That is why many primitive defining functions were developed, such as planes, LP distance metrics [1], Super-quadrics [2, 3], Generalized distance functions [4], Skeletal primitives [5], Sweep objects [6], which are viewed as primitive implicit surfaces. Depending on the value of C, implicit surface modeling is divided into three categories as follows:

- Zero implicit surface $fi(x,y,z)=0$ [7, 8, 9, 10, 11, 12, 13].
- Constructive geometry $fi(x,y,z)=1$ [2, 14, 15].
- Soft object modeling $fi(x,y,z)=0.5$ [15, 16, 17, 18].

Furthermore, for creating a complex surface blending operation was also proposed to connect primitive implicit surfaces smoothly using automatically generated transition. In the literature, a lot of blending operations have been developed and they are categorized as follows:

- Blending operations without blending range control [19], which deform entire blended surfaces after blending.
- Blending operations with blending range parameters [7, 8], which deform blended surfaces locally after blending.
- Blending operations with C^1 continuity for generating sequential blending operations [15, 20]. Blends with transition of high order continuity were proposed in [21].

In soft object modeling, field functions are used as a defining function to define a soft object. Thus, soft objects are allowed to be blended by Soft blend, which need summation operation only and has lower computing complexity. In addition, field functions with adjustable inner and outer influential radii [22] were also proposed to allow soft blend to have blending range control parameters.

However, existing blending operations mentioned above still face a difficulty that: the shape of the contour of the automatically generated transition of a union blending surface is always similar to that of the contour of blended surfaces. For example, when a cylinder is blended with a cube, the contour of the transition is always like a circle. To conquer the difficulty and enable one to freely determine the shape of the contour of the transition of a union, this paper proposes variable influential method for soft object modeling. This method allows one to freely choose an implicit surface. And then the chosen surface is applied to control and determine the shape of the contour of the automatically generated transition of a union blending surface in soft object modeling.

The method is described briefly as follows. For any union blending surface on soft objects,

- Freely choose an implicit surface, which is called contour surface and is defined using a ray-linear function.
- The chosen contour surface varies the influential radius of the field functions used to define blended soft objects, such that the contour's shape of the transition of the union blend can be like that of the chosen contour surface.

This paper is organized as follows. Implicit surface is defined in Section II. The purpose of this paper is described in Section III. Variable influential radius method is presented in Section IV. Some demonstrations are shown in Section V. Conclusion is given in Section VI.

II. Definition of Implicit Surface

A. Implicit Surface

An implicit surface is defined as a level surface of a defining function by

$$\{(x,y,z) \in R^3 \mid fi(x,y,z)=C\},$$

where $fi(x,y,z){:}R^3 \rightarrow R$ is called defining function, C is a fixed constant which is usually set by 0, 0.5 or 1. Moreover, an implicit surface is usually viewed as an implicit solid for further processing by booolean set operations. An implicit solid is defined as a half space by

$$\{(x,y,z) \in R^3 \mid fi(x,y,z) \le C\}.$$

Precisely speaking, when viewed as an implicit solid, $fi(x,y,z)$ must satisfy the following conditions:

- Every surface $fi(x,y,z)=t$ for $t \in R$ is an implicit surface.
- $fi(x,y,z)$ is an inside-outside function, which means $fi(x,y,z)=C$ divides the space into three regions: $fi(x,y,z)=C$ stands for the boundary of the object; $fi(x,y,z)<C$ the inside; and $fi(x,y,z)>C$ the outside.

Depending on the value of C: 0, 1 or 0.5, implicit surface modeling is divided into the following categories:

- Zero implicit surface $fi(x,y,z) \le 0$, whose advantage is the compliment of $fi(x,y,z) \le 0$ is obtained by $-fi(x,y,z) \le 0$.
- Constructive geometry $fi(x,y,z) \le 1$, whose advantage is the compliment of $fi(x,y,z) \le 1$ is obtained by $1/fi(x,y,z) \le 1$.
- Soft object modeling $fi(x,y,z) \ge 0.5$. Because field functions are used as defining functions $fi(x,y,z)$ in soft object modeling, soft objects are blended easily by summation operation, called soft blend.

For simplicity and uniformity, in the following implicit surface (solid) is denoted by

$$fi(x,y,z)=0,\ fi(x,y,z)=0.5 \text{ and } fi(x,y,z)=1,$$

for zero implicit surface, soft objects and constructive geometry, respectively.

B. Existing Defining Functions

Defining functions decide the shapes of primitive implicit surfaces to be further blended for generating a complex implicit surface, so a lot of defining functions were proposed and they are listed as follows:

- Plane:

$$f(x,y,z)=|[x,y,z]\bullet \boldsymbol{n}|/a, \quad (1)$$

where $\boldsymbol{n}$ is a given unit normal vector toward the plane and parameter $a>0$ controls the shortest Euclidean distance from the plane to the origin.

- L_P distance metrics [1]:

$$f(x,y,z)=(|x/a|^n+|y/b|^n+|z/c|^n)^{1/n}, \quad (2)$$

where parameters a, b and c decide the axial lengths of x, y, and z of the shape $f(x,y,z)=C$

- Super-quadrics [2, 3]:

$$f(x,y,z)=((|x/a|^{n1}+|y/b|^{n1})^{n2/n1}+|z/c|^{n2})^{1/n2}, \quad (3)$$

where curvature parameters $n_1>1$ and $n_2>1$ control the shape of $f(x,y,z)=C$.

- Generalized distance functions [4]:

$$f(x,y,z)=(\textstyle\sum_{i=1}^{k} |[x,y,z]\bullet \boldsymbol{n}_i|^n)^{1/n}, \quad (4)$$

where $\boldsymbol{n}_i$, $i=1, \ldots, k$, are unit normal vectors to the planes $|[x,y,z]\bullet \boldsymbol{n}_i|=C$ and curvature parameter $n>1$ controls the shape $f(x,y,z)=C$.

- Skeletal primitives [5]:

$$f(x,y,z)=r/I_r, \quad (5)$$

where r is the shortest distance from the point (x,y,z) to a given 3D line segment, and I_r is a given influential radius.

- Sweep objects [6].

Some implicit surfaces defined using the above functions are shown in Figure 1.

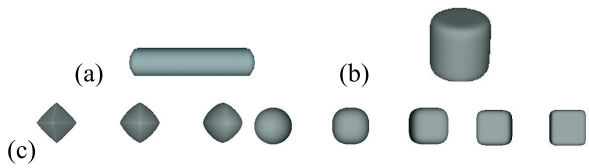

Figure 1. (a). Skeletal primitive. (b). Shape of super-quadrics. (c). Shapes of L_P distance metrics with parameter n varying from 1.1, 1.3, 1.5, 2, 3, 5, 10, 20.

C. Blending Operations

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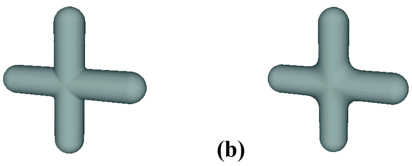
In fact, planes, spheres, super-ellipsoids and cylinders in Subsection A can further be used as primitives $f_i(x,y,z)=C$, $i=1,\ldots,k$, to define a more complex implicit surface through a blending operator $B_k(x_1,\ldots,x_k):R^k\rightarrow R$ by

$$B_k(f_1(x,y,z),\ldots,f_k(x,y,z))=C$$

where $B_k(f_1,\ldots,f_k)=C$ is called blending surface and $B_k(f_1,\ldots,f_k)$ is called blending operation or blend. $B_k(f_1,\ldots,f_k)=C$ connects surfaces $f_i(x,y,z)=C$ smoothly by automatically generated transition which is tangent to every $f_i(x,y,z)=C$. For example, Figure 2(a) shows a pure union of two cylinders which has sharp edges, but Figure 2(b) shows a union blending operation of two cylinders which is connected smoothly with automatically generated transition.



(a) (b)

Figure 2. (a). A pure union of cylinders with sharp edge. (b). A union blend of cylinders with smooth transition generated automatically.

In the literature, many blending operations have been developed and they are categorized and introduced, respectively, in Subsections II.C.1-5.

1. Blending operations without blending range control

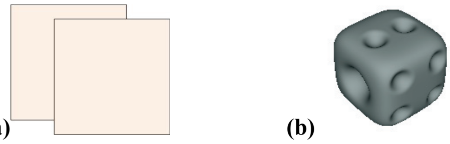
In constructive geometry [19], boolean set operations on surfaces $f_i(x,y,z)=1$ were proposed and they are presented as follows:

- Super-ellipsoidal intersection:
$$B_{Sk}(x_1,\ldots,x_k)=(x_1^{\,n}+\ldots+x_k^{\,n})^{1/n}, \quad (6)$$
- Super-ellipsoidal union :
$$B_{Sk}(x_1,\ldots,x_k)=(x_1^{\,-n}+\ldots+x_k^{\,-n})^{-1/n}.$$
- Super-ellipsoidal difference :
$$B_{Sk}(x_1,\ldots,x_k)=(x_1^{\,-n}+x_2^{\,n}\ldots+x_k^{\,n})^{-1/n}.$$

Because they do not offer blending range parameters, blended surfaces are always deformed totally after bleding, as shown in Figure 3.

2. Blending operations with blending range control

In zero implicit surface [7, 8], Hoffman proposed a union blending operator on surfaces $fi(x,y,z)=0$ by



(a) (b)

Figure 3. (a). Parallel planes. (b). Difference of a super-ellipsoidal intersection of three pairs of parallel planes from 21 spheres, where planes are deformed totally because of lacking adjustable blending range parameters.

$$B_H(x_1,x_2)=r_2^{\,2}x_1^{\,2}+r_1^{\,2}x_2^{\,2}+r_2^{\,2}r_1^{\,2}-2r_2^{\,2}r_1x_1-2r_1^{\,2}r_2x_2+2px_1x_2, \quad (7)$$

where p is a curvature parameter and $-\infty<p<r_2r_1$, and r_1 and r_2 are blending range parameters.

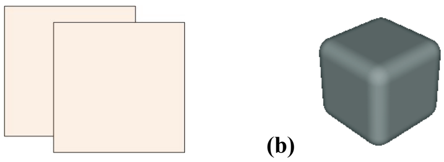
In addition, intersection and difference operations are obtained from the dual of $B_H(f_1,f_2)$ by

$$-B_H(-f_1,-f_2) \text{ and } -B_H(-f_1,f_2).$$

Because $B_H(x_1,x_2)$ offer blending range parameters r_1 and r_2, the trasition of the blending surface is generated within the blending regions:

- $0\le f_1(x,y,z)\le r_1$ and $0\le f_2(x,y,z)\le r_2$ for union on surfaces $f_i(x,y,z)=0$,
- $-r_1\le f_1(x,y,z)\le 0$ and $-r_2\le f_2(x,y,z)\le 0$ for intersection on surfaces $f_i(x,y,z)=0$,

and hence blended primitive surfaces are deformed locally after blending as shown in Figure 4.



(a) (b)

Figure 4. (a). Parallel planes. (b). Hoffman's intersection of three pairs of parallel planes, which deforms locally due to blending range parameters applied.

3. Blending operations with C^1 continuity for generating sequential blending operations

This kind of blending operations has been developed from the displacement method [13] and the scale method [15]. The scale method is described as follows:

- By using an existing union operator $B_{Ak}(x_1,\ldots,x_k)$ on surfaces $f_i(x,y,z)=1$ of constructive geometry, the scale method develops a scale union operator $B_{SAk}:R^n\rightarrow R_+$, where $R_+\equiv\{x\ge 0|\, x\in R\}$, for sequential unions by

$$B_{SAk}(x_1,\ldots,x_k)=\begin{cases} Min(x_1,\ldots,x_k) & Min(x_1,\ldots,x_k)\le 0 \\ h & \text{otherwise} \end{cases},$$

where h is the root of the equation $B_{Ak}(x_1/h,\ldots,x_k/h)=1$.

- By using an existing intersection operator $B_{Sk}(x_1,\ldots,x_k)$ on surfaces $f_i(x,y,z)=1$, the scale method develops a scale intersection operator $B_{SSk}:R^n\rightarrow R_+$ for sequential intersection operations by

$$B_{SSk}(x_1,\ldots,x_k)=\begin{cases} Max(x_1,\ldots,x_k) & Max(x_1,\ldots,x_k)\le 0 \\ h_p & \text{otherwise} \end{cases},$$

where h is the root of the equation $B_{Sk}(x_1/h,\ldots,x_k/h)=1$.

By using $B_H(x_1,x_2)$ as B_{Ak} and B_{Sk}, blending operators developed from the scale method are presented as follows:

- When $B_H(x_1\text{-}1,x_2\text{-}1)=0$ in Equation (7) is used as $B_{A2}(x,x_2)$ in B_{SAk}, a scale union operator B_{SA2} with blending range parameters r_1 and r_2 for sequential unions is given by

$$B_{SA2}(x_1,x_2)=\begin{cases} (-b\pm(b^2-4ac)^{0.5})/(2a) & \text{if } a\neq 0 \text{ and region III} \\ -c/b & \text{if } a=0 \text{ and region III} \\ Min(x_1,x_2) & \text{otherwise} \end{cases},$$

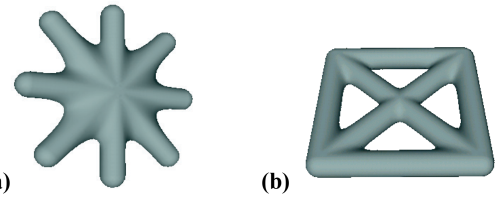
where $a=(1+2r_2)r_1^2+(1+r_1)^2r_1^2+2p$, $b=-2(x_2(1+r_2)r_1^2+x_1(1+r_1)r_2^2+(x_1+x_2)p)$ and $c=r_1^2x_2^2+r_2^2x_1^2+2px_1x_2$.

- When $B_H(1\text{-}x_1,1\text{-}x_2)=0$ in Equation (7) is used as $B_{S2}(x_1,x_2)$ in B_{SSk}, a scale intersection operator B_{SS2} with blending range parameters r_1 and r_2 for sequential intersections is given by

$$B_{SS2}(x_1,x_2)=\begin{cases} (-b\pm(b^2-4ac)^{0.5})/(2a) & \text{if } a\neq 0 \text{ and region III} \\ -c/b & \text{if } a=0 \text{ and region III} \\ Max(x_1,x_2) & \text{otherwise} \end{cases}, \quad \textbf{(8)}$$

where $a=(1\text{-}2r_2)r_1^2+(1\text{-}r_1)^2r_1^2+2p$, $b=-2(x_2(1\text{-}r_2)r_1^2+x_1(1\text{-}r_1)r_2^2+(x_1+x_2)p)$ and $c=r_1^2x_2^2+r_2^2x_1^2+2px_1x_2$.

Here, Region **III** stands for $\{(x_1,x_2)\in R_+^2|x_1/(1+r_1)<x_2<(1+r_2)x_1\}$ for $B_{SA2}(x_1,x_2)$, and $\{(x_1,x_2)\in R_+^2|(1\text{-}r_1)x_2<x_1$ and $x_2>(1\text{-}r_2)x_1\}$ for $B_{SS2}(x_1,x_2)$. Figure 5(a) demonstrates sequential union blends of four intersected cylinders by $B_{SA2}(B_{SA2}(B_{SA2}(f_1,f_2),f_3),f_4)=1$ and Figure 5(b) sequential unions of six end-to-end connected cylinders by $B_{SA2}(B_{SA2}(B_{SA2}(B_{SA2}(B_{SA2}(f_1,f_2),f_3),f_4),f_5),f_6)=1$.



(a) (b)

Figure 5. (a). Sequential unions of four intersecting cylinders of constructive geometry with blending range control using $B_{SA2}(x_1,x_2)$. (b). Sequential unions of six end-to-end connecting cylinders of constructive geometry using $B_{SA2}(x_1,x_2)$.

4. Soft blend

Soft object modeling is a special version of implicit surface modeling and a soft object is defined as the point set $\{(x,y,z)\in R^3|f(x,y,z)=0.5\}$. Especially, defining function $f(x,y,z)$ is a field function defined by

$$f(x,y,z)=(P\circ d)(x,y,z)=P(d((x,y,z)), \quad (9)$$

where $P(d):R_+\rightarrow[0,1]$ is called potential function, $d(x,y,z):R^3\rightarrow R_+$ is called distance function, and $R_+\equiv\{x\ge 0|x\in R\}$.

In addition, as shown in Figure 6(a), distance function $d(x,y,z)$ is created by using a closed surface as its influential region and is defined by

$$d(x,y,z)=\overline{cX}/\overline{cI}=r/I_r, \quad (10)$$

where r is $(x^2+y^2+z^2)^{0.5}$ and $I_r=\|\overline{cI}\|$ is called influential radius, which is the distance from origin c to the intersecting point of $\overline{cX}$ with the boundary of the closed surface. In fact, all the defining functions stated in Subsection I.2 are suitable to be a distance function $d(x,y,z)$ because they can be reformulated to be Equation (10) due to their ray-linear property. In addition, potential function $P(d)$ must satisfy the following conditions:

- $P(0.5)=0.5$, $P'(0)=0$ and $P'(1)=0$.
- As the value of d increases from 0 to 1, the value of $P(d)$ drops from 1 to 0.

These conditions are obeyed by Blanc's potential function $P(d)$ in [1] described as follows:

$$P(d)=\begin{cases} 1-(3d^2)^2/(s+(4.5-4s)d^2) & d\le 0.5 \\ (1-d^2)^2/(0.75-s+(1.5+4s)d^2) & 0.5<d\le 1 \\ 0 & d>1 \end{cases}. \quad (11)$$

where s is softness parameter and the shape of $p=P(d)$ is shown in Figure 6(a).

Because of the conditions of $P(d)$, as shown in Figure 6(b)

- The union of soft objects $f_i(x,y,z)=0.5$, $i=1\ldots,k$, is obtained using soft blend operator $B_k(x_1,\ldots,x_k)=x_1+\ldots+x_k$ by

$$B_{Sk}(f_1,\ldots,f_k)=f_1(x,y,z)+f_2(x,y,z)+\ldots+f_k(x,y,z)=0.5, \quad (12)$$

- The shape of soft object $f(x,y,z)=(P\circ d)(x,y,z)=0.5$ is the same as the shape of $d(x,y,z)=0.5$, half the size of the closed surface $d(x,y,z)=1$.
- The shape of the closed surface $d(x,y,z)=1$ determines the shape of soft object $f(x,y,z)=(P\circ d)(x,y,z)=0.5$.
- For any union blend on soft objects, the automatically generated tranition always lies within the blending region:

$$0.5\le d(x,y,z)\le 1, \text{ i.e. } 0\le f(x,y,z)\le 0.5.$$

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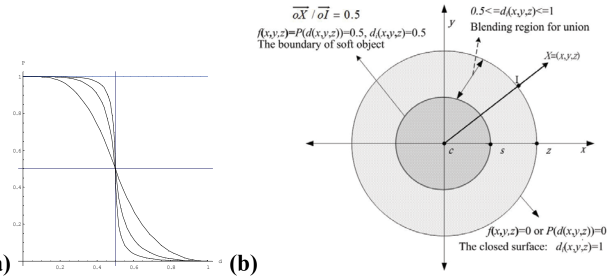
In addition, union and intersection operations of constructive geometry in Subsections C.1 and C.3 also work for blending soft objects, but union becomes intersection and intersection becomes union.



(a) (b)

Figure 6. (a). The shape of Blanc potential function *p*=*P*(*d*) with different values of parameter *s*. (b). The shape of a soft object of a sphere *f*(*x,y,z*)=0.5, the same as *d*(*x,y,z*)=0.5 shown in heavy shaded area; The light shaded area shows the blending region 0.5≤*d*(*x,y,z*)≤1 of a spherical soft object of a union.

5. Field functions with adjustable inner and outer influential radii

In fact, every soft object of a soft blend always has a fixed blending range 0.5, i.e. $0.5\le f(x,y,z)\le 0$, as shown in Figure 7(a). As a result,

- Every soft object always has a fixed inner influential radius $I_r/2=|\overrightarrow{sc}|$ and an outer influential radius $I_r/2=|\overrightarrow{sz}|$.
- The sizes of inner and outer radii always depend on the size of soft object. That is, a large soft object has a large blending region and a small one a small blending region. This is seen in Figure 7(c), where the large cylinder has a larger blending region.

To offer soft blend offer blending range parameters, field functions with adjustable inner and outer influential radii were proposed in [22], which provides parameters w_2 and w_1 to adjust the inner and the outer radii of influence by $w_2I_r/2$ and $w_1I_r/2$, respectively. One of the field functions is introduced as follows:

$$f(x,y,z)=(P\circ T\circ d)(x,y,z), \qquad (13)$$

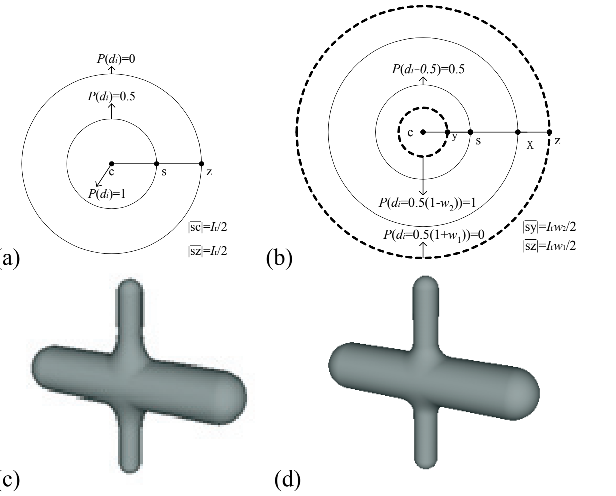
where $$T(d)=\begin{cases} 1 & d>0.5(1+w_1) \\ (d-0.5)/w_1+0.5 & 0.5\le d\le 0.5(1+w_1) \\ N_n(d)=Ad^2+Bd+C & 0.5(1-w_2)\le d<0.5 \\ 0 & \text{otherwise} \end{cases},$$

and $w_2\le 2w_1$, $w_2\le 1$ and $w_1>0$; $A=2(1/w_1-1/w_2)/w_2$, $B=1/w_1-A$ and $C=0.5-0.5B-0.25A$.

Thus, when $(P\circ T\circ d)(x,y,z)$ in Eq. (13) is applied as a field function to define a soft object, the blending region becomes:

- $0.5\le d(x,y,z)\le 0.5(1+w_1)$ for a union blend, i.e. the outer influential radius is $w_1I_r/2$,
- $0.5(1-w_2)\le d(x,y,z)\le 0.5$. for an intersection, i.e. the inner influential radius is $w_2I_r/2$,

, as shown in Figure 7(b). This shows that w_1 and w_2 are able to be used as blending range parameters for a union and an intersection blends, respectively. Especially, w_1 and w_2 enable a large object to have a small blending region and a small one a large blending region. These are seen in Figure 7(d), where the large cylinder has a smaller blending region.



(a) (b) (c) (d)

Figure 7. (a). Inner radius $I_r/2=|\overrightarrow{sc}|$ and outer radius $I_r/2=|\overrightarrow{sz}|$ of the field function (*P* ∘ *d*)(*x,y,z*) of a sphere. (b). Inner radius $w_2I_r/2=|\overrightarrow{sy}|$ and outer radius $w_1I_r/2=|\overrightarrow{sz}|$ of field function (*P* ∘ *T* ∘ *d*)(*x,y,z*) of a sphere, where w_1 and w_2 are adjustable parameters such that the influential region becomes bounded by broken curves. (c). Soft blend of a small and a large cylinders, where the large one always has a larger blending region. (d). Soft blend of a small and a large cylinders, where the large one can have a smaller blending region because of using Equation (13) with w_1=0.5 as its field function.

III. Purpose of the Paper

A. Difficulty of Existing Blends

Regarding a union blending operator $B_2(x_1,x_2)$ on zero implicit surfaces $f_1(x,y,z)=0$ and $f_2(x,y,z)=0$, the shape of $B_2(x_1,x_2)=0$ is required to be arc-shaped like the dotted curve in Figure 8(a). Let $H_2(x_1,x_2)=0$ denote the arc part of the curve $B_2(x_1,x_2)=0$. Then every point (x_1,x_2) on curve $H_2(x_1,x_2)=0$ generates a 3D curve, i.e. the intersection of surfaces $f_1(x,y,z)=x_1$ and $f_2(x,y,z)=x_2$. All the intersection curves constitute the smooth transition of blending surface $B_2(f_1,f_2)=0$. This implies that for a union blend $B_2(f_1,f_2)=0$:

- The smooth transition of $B_2(f_1,f_2)=0$ is given by

$$\{ f_1(x,y,z)=x_1 \cap f_2(x,y,z)=x_2 \mid H_2(x_1,x_2)=0 \text{ and } (x_1,x_2)\in R^2\},$$

where the symbol $\cap$ denotes intersection.

- The transition is bounded within the blending region:

$$\{ (x,y,z)\in R^3 \mid 0\le f_1(x,y,z)\le r_1 \text{ and } 0\le f_2(x,y,z)\le r_2 \}.$$

- Iso-surfaces $f_1(x,y,z)=i$, $0\le i\le r_1$, and $f_2(x,y,z)=i$, $0\le i\le r_2$, are used to generate the transition of $B_2(f_1,f_2)=0$, whose contour hence is like $f_1(x,y,z)=r_1$ or $f_2(x,y,z)=r_2$.

From the union blending curves $B_2(x_1,x_2)=0.5$ of soft object modeling in Figure 8(b), for a soft blend $B_2(f_1,f_2)=0.5$ on soft objects $f_1(x,y,z)=(P\circ d_1)=0.5$ and $f_2(x,y,z)=(P\circ d_2)=0.5$, the following are obtained similarly:

- The transition of blending surface $B_2(f_1,f_2)=0.5$ is given by

$$\{d_1(x,y,z)=x_1 \cap d_2(x,y,z)=x_2 \mid B_2(x_1,x_2)=0.5 \text{ and } (x_1,x_2)\in R^2\}.$$

- The transition is bounded within the blending region:

$$\{(x,y,z)\in R^3 \mid 0\le f_1(x,y,z)\le 0.5 \text{ and } 0\le f_2(x,y,z)\le 0.5 \}, \text{ i.e.}$$
$$\{(x,y,z)\in R^3 \mid 0.5\le d_1(x,y,z)\le 1 \text{ and } 0.5\le d_2(x,y,z)\le 1 \}.$$

- Iso-surfaces $d_1(x,y,z)=i$, $0.5\le i\le 1$, and $d_2(x,y,z)=i$, $0.5\le i\le 1$, are used to generate the transition of $B_2(f_1,f_2)=0.5$, whose contour is hence like $d_1(x,y,z)=1$ or $d_2(x,y,z)=1$.

Because iso-surfaces of a defining function or a distance function always have similar shapes, this causes that all the blending operations in zero implicit surfaces and even in soft object modeling face a difficulty that The contour of the resulting transition of a union blending surface always has a similar shape with the contour of blended surfaces. This phenomenon is seen in Figure 9, where Figures 9(c) and 9(e) show similar transition's contours with the blended surface's contours.

B. Paper's Purpose

To conquer the difficulty of existing blending operations stated in Section III.A, this paper proposes variable influential radius method for solving the difficulty of soft object modeling. Based on this method, for any union blending surface on soft objects, one can

- Freely assign an implicit surface as a contour surface.
- Then the chosen contour surface is used to vary the outer influential radius of the field functions used to define blended soft objects, such that the contour's shape of the transition of the resulting union blending surface is similar to that of the chosen contour surface.

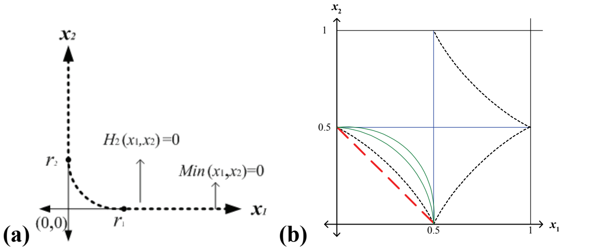


Figure 8. (a). The shape of a union blending curve $B_2(x_1,x_2)=0$ on zero implicit surfaces, dotted curve. (b). The shapes of a union blending curve $B_2(x_1,x_2)=0.5$ on soft objects, the curves lie within the region $0<x_1<0.5$ and $0<x_2<0.5$.

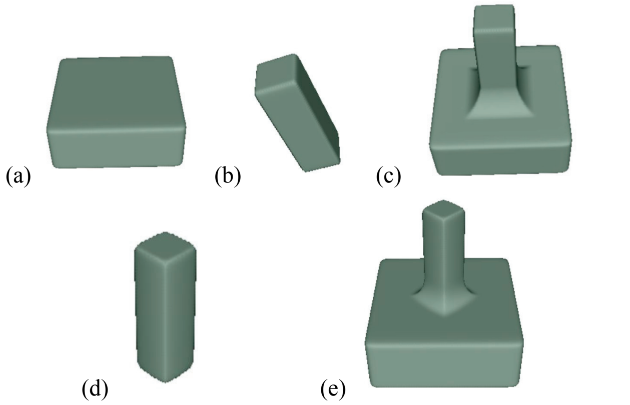


Figure 9. (a). Cube. (b). Cube. (c). Union of the objects in Figures. 9(a)-(b), where the contour of the transition is similar to that of Figure 9(b). (d). Prism. (e). The union of the objects in Figures 9(a) and 9(d), where the contour of the transition is similar to that of Figure 9 (d).

Figure 10(b) shows a union blend of the two cubes in Figure 10(a) and the shape of the transition is similar to that of the right object in Figure 10(a). But when variable influential radius method is applied and surfaces like a pinched prism, a prism, a circle and a cube in Figures 10(c)-(f) are assigned as the contour surfaces, the resultant union blending surfaces are listed in Figures 10(g)-(j), where transition's contours are similar to those in Figures 10(c)-(f).

IV. Variable Influential Radius Method

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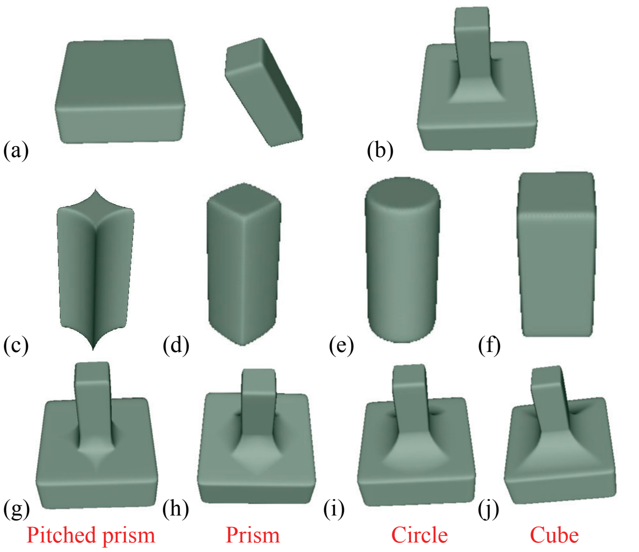

Figure 10. (a). a cube and a rectangular bar. (b). Blending surface of an existing union on the objects in Fig. 10(a). (c)-(f). Chosen contour surfaces to be put into newly proposed blending operations. (g)-(j). Blending surfaces of new union operations which generate a transition with contour like the chosen contour surfaces in Figures. 10(c)-(f).

This section introduces variable influential radius method, which needs to choose contour surface $f_c(x,y,z)=1$ where $f_c(x,y,z)$ needs to be a ray-linear function.

A. Ray-linear Function

Non-negative ray-linear property is introduced first and a related theorem is described then.

- **Definition 1:** A function $f(x,y,z):R^3 \to R_+$ is called non-negative ray-linear if $f(ax, ay, az)=af(x,y,z)$ holds for any $(x,y,z) \in R^3$ and $a \in R_+$ [14].

From **Definition 1**, **Theorem 1** was proposed in [15] and is described below:

- **Theorem 1:** If $f(x,y,z):R^n \to R_+$ is non-negative ray-linear, then $f(x,y,z)$ can be reformulated to r/I_{rf}, where $r=(x^2+y^2+z^2)^{0.5}$ and I_{rf} is the influential radius for (x,y,z) with respect to the closed surface $f(x,y,z)=1$, viewed as the influential region. That is, a ray-linear function $f(x,y,z)$ is a distance function $d(x,y,z)$ in Equation (10), too.

B. Variable Influential Radius Method

For any union blend $B_2((P \circ d_1)(x,y,z), (P \circ d_2)(x,y,z))=0.5$ in soft object modeling, variable influential radius method attains the purpose stated in Section III.B through the following three steps:

- Step 1: Choose a contour surface $f_c(x,y,z)=1$ where $f_c(x,y,z)$ is ray-linear.
- Step 2: Replace $B_2((P \circ d_1)(x,y,z), (P \circ d_2)(x,y,z))=0.5$ with

$$B_2((P \circ T \circ d_1)(x,y,z), (P \circ d_2)(x,y,z))=0.5. \qquad (14)$$

- Step3: Replace w_1 of $T(d)$ of $(P \circ T \circ d_1)(x,y,z)$ with

$$w_1=2d_1(x,y,z)/f_c(x,y,z)-1. \qquad (15)$$

In Equation (14), the blending region of $(P \circ T \circ d_1)(x,y,z)=0.5$ lies in $5 \le d_1(x,y,z) \le 0.5(1+w_1)$ for a union blend. Besides, setting $2d_1(x,y,z)/f_c(x,y,z)$**-**1 in Equation (15) to parameter w_1 of $T(d)$ for any (x,y,z) enables that the blending region of $(P \circ T \circ d_1)(x,y,z)=0.5$, i.e. $0.5 \le d_1(x,y,z) \le 1$, in Equation (14) is equivalent to the region

$$\{(x,y,z) \in R \,|\, 0.5 \le d_1(x,y,z) \le 0.5(2d_1(x,y,z)/f_c(x,y,z)-1+1)\}, \text{ i.e. } \{(x,y,z) \in R^3 \,|\, 0.5 \le d_1(x,y,z) \text{ and } f_c(x,y,z) \le 1\}.$$

The region above means that the contour of the transition of the union blend becomes like the shape of the contour surface $f_c(x,y,z)=1$, fulfilling the purpose stated in Section III.B.

Explain Equation (15) according to the ray-linear property. Substituting $d_1(x,y,z)=r/I_{rd}$ and $f_c(x,y,z)= r/I_{rf}$ into Equation (15) gives

$$w_1 \text{ of } T(d)=2d_1(x,y,z)/f_c(x,y,z)\mathbf{-1=}\ 2I_{rf}/I_{rd}-1.$$

This tells that for any (x,y,z) the outer influential radius of $(P \circ T \circ d_1)(x,y,z)$ changes from $I_{rd}/2$ to

$$I_{rf} - I_{rd}/2,$$

which indicates that Equation (15) varies the outer influential radius of the field functions of blended soft objects by the influential radius I_{rf} of the contour surface $f_c(x,y,z)=1$. This is described in 2D space in Figure 11. Figure 11(a) shows the blending region of $(P \circ d_1)(x,y)=0.5$ and Figure 11(b) shows the blending region of $(P \circ T \circ d_1)(x,y)=0.5$ after w_1 is given by Equation (15), where the influential radius $I_{rd}/2$ changes from $I_{rd}/2$ to $I_{rf} - I_{rd}/2$.

V. Demonstration of Variable Influential Radius Method

This section demonstrates some union blending surface generated using variable influential radius.

A. Soft Blend with Variable Influential Radius

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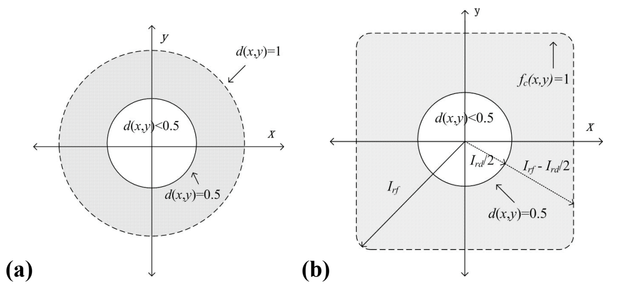

Figure 11. (a). Dark region shows the blending region of ($P \circ d_1$)(*x*, *y*)=0.5 of a circle. (b). Dark region shows the blending region of ($P \circ T \circ d_1$)(*x*, *y*)=0.5 of a circle after parameter w_1 is varied by Eq. (15) where $f_c(x,y)$=1 is a super-elliptic contour curve.

Figure 12(a) displays a cube and a prism-shaped bar, which are soft object defined in the following:

- Prism-shaped bar: $f_1(x,y,z)=(P \circ d_1)(x,y,z)=0.5$, where $P(d)$ is in Equation (11) with s=0 and $d_1(x,y,z)$ is a super-quadrics in Equation (3) written by

$$d_1(x,y,z)=((|x/8|^{n1}+|y/8|^{n1})^{20/n1}+|z/24|^{20})^{1/20}, \qquad (19)$$

- Cube: $f_2(x,y,z)=(P \circ d_2)(x,y,z)=0.5$ in Eq. (9), where $P(d)$ is in Eq. (11) with s=0 and

$$d_2(x,y,z)=(|x/30|^{20}+|y/30|^{20}+|z/12|^{20})^{1/20}. \qquad (20)$$

Thus, soft blend $Bs_2(f_1,f_2)=f_1(x,y,z)+f_2(x,y,z)=0.5$ by Equation (12) generates a union blending surface in Figure 12(b) whose transition's contour is always similar to that of the prism-shaped bar in Figure 12(a).

Now, variable influential radius method in Equation (14) is applied to soft blend $Bs_2(f_1,f_2)$=0.5 above. That is, $f_1(x,y,z)$ is replaced with $(P \circ T \circ d_1)(x,y,z)=0.5$, where $P(d)$ is in Equation (11) with s=0, $T(d)$ in Equation (13) with w_1=1 and w_2=1. In addition, $f_c(x,y,z)$ given, respectively, by $f_c(x,y,z)=$

$$d_1(x,y,z)=1 \text{ in Eq. (19) but } n_1 = 0.6, 1.2, 1.5, 2, 3, \text{ and } 20,$$

are used as contour surfaces. The shapes of the chosen contour surfaces are displayed in Figures 12(c)-(h), respectively. Figures 12(i)-(n) display the union blending surfaces of soft objects $Bs_2(f_1,f_2)=f_1(x,y,z)$=0.5 after performing variable outer influential radius with contour surfaces stated above. These shapes indicate that the contour's shapes of the transitions of the resulting union blending surfaces are similar to those of the chosen contour surfaces in Figures 12(c)-(h).

Figure 13(a) demonstrates two soft objects, a cube $f_2(x,y,z)$=0.5 and a rose-shaped bar, $f_1(x,y,z)$=0.5, whose definition be found in [23]. Performing soft blend $Bs_2(f_1,f_2)= f_1(x,y,z)+f_2(x,y,z)$=0.5 by Equation (12) on them generates a blending surface in Figure 13(b) whose transition's contour

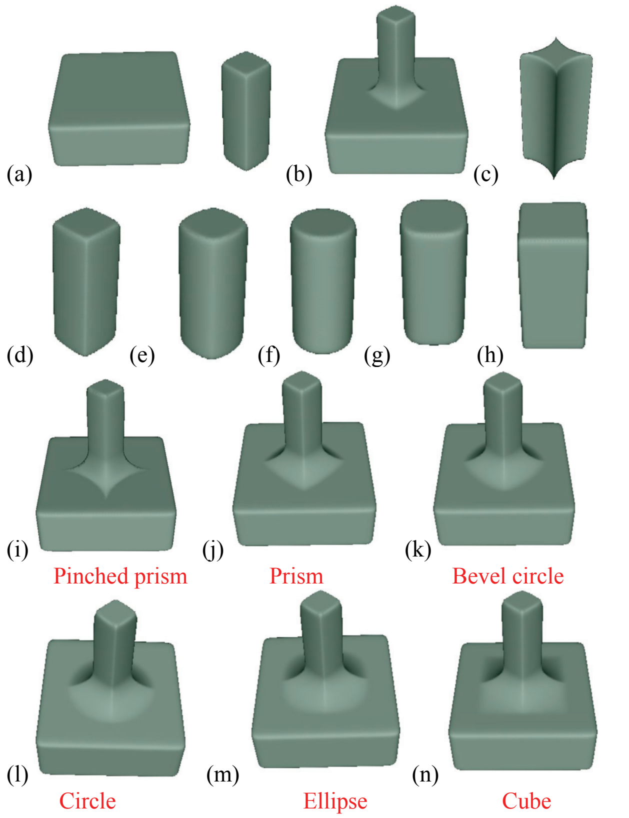

Figure 12. (a). Cube and prism. (b). Blending surface of a soft blend on the objects in Figure 12(a). (c)-(h). Chosen contour surfaces for variable influential radius method. (i)-(n). Union blending surfaces of soft blend $Bs_2(f_1,f_2)$=0.5 after performing variable influential radius method using the contour surfaces in Figures 12(c)-(h).

is similar to that of the rose-shaped bar in Figure 13(a). However, after variable influential radius in Eq. (14) is applied to soft blend $Bs_2(f_1,f_2)$=0.5 and the star-shaped object, whose definition can be found in [23], in Figure 13(c) is used as the contour surface $f_c(x,y,z)$=1, Figure 13(d) displays the resultant blending surface of the soft blend $Bs_2(f_1,f_2)$=0.5, which indicates the contour's shape of the transition of the blending surface is similar to that of the chosen contour surface in Figure 13(c).

B. Scale Union with Variable Influential Radius

This case was shown in Section III.B and is discussed below. Figure 14(a) displays a scale union of a cube and a rectangular bar, which are soft object defined below:

- Rectangular bar: $f_1(x,y,z) = (P \circ d_1)(x,y,z)$=0.5 in Equation

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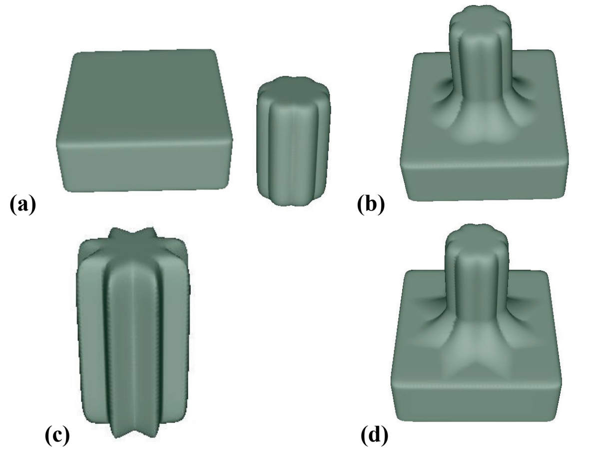

Figure 13. (a). A cube and a rose-shaped objects. (b). Soft blend of the objects in Figure 13(a). (c) Star-shaped bar. (d). Soft blend $Bs_2(f_1,f_2)$=0.5 in (a) after performing variable influential radius method with the object in (c) as the contour surface.

(9), where $P(d)$ is in Equation (11) with s=0 and $d_1(x,y,z)$ is a super-quadrics in Equation (2), defined by

$$d_1(x,y,z)=((|x/8|^{20}+|y/8|^{20}+|z/24|^{20})^{1/20},$$

- Cube: $f_2(x,y,z)=(P \circ d_2)(x,y,z)=0.5$ in Equation (9), where function $P(d)$ is in Eq. (11) with s=0 and $d_1(x,y,z)$ is a cube in Equation (2) by

$$d_2(x,y,z)=(|x/30|^{20}+|y/30|^{20}+|z/12|^{20})^{1/20}.$$

Thus, scale union $Bss_2((P \circ d_1)(x,y,z), (P \circ d_2)(x,y,z))=0.5$ on the two soft objects above by Equation (8) generates the blending surface shown in Figure 14(b), whose transition's contour is similar to that of the Rectangular bar in Figure 14(a). Now, variable influential radius method in Equation (15) is applied to the scale union, i.e. $Bss_2((P \circ T \circ d_1)(x,y,z), (P \circ d_2)(x,y,z))=0.5$ and $T(d)$ in Equation (13) with w_1=1 and w_2=1. In addition, contour surface $f_c(x,y,z)$=1 is assigned by $f_c(x,y,z)$=

$$d_1(x,y,z)=((|x/13|^{n1}+|y/13|^{n1})^{20/n1}+|z/24|^{20})^{1/20}=1,$$

where n_1 is set 0.6, 1.1, 1.5, 2, 3, and 20, respectively, and whose contour's shapes are displayed in Figures 14(c)-(h), respectively. Figures 14(i)-(n) show the scale union blending surfaces of $Bss_2(f_1,f_2)$=0.5 after performing variable influential radius method. These shapes indicate that their contour's shapes of the transitions are similar to those of the chosen contour surfaces in Figures 14(c)-(h).

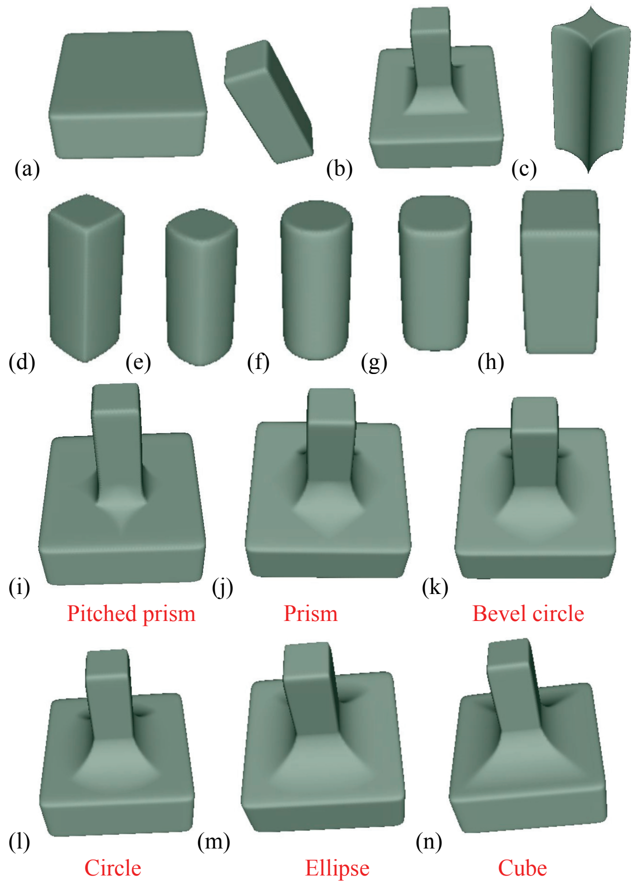

Figure 14. (a). A cube and a rectangular bar. (b). Blending surface of a scale union on the objects in Figure 14(a). (c)-(h). Chosen contour surfaces for variable outer influential radius method. (i)-(n). Blending surfaces of scale union $Bss_2(f_1,f_2)$=0.5 after being applied variable outer influential radius method with the contour surfaces in Figures 14(c)-(h).

VI. Conclusion

Existing union blending operations of implicit surface modeling face a difficulty that the shape of the contour of the automatically generated transition is always similar to the shape of the contour of blended surfaces. To conquer the difficulty, this paper has successfully developed variable influential radius method for union blending surfaces of soft object modeling. In this method, one can freely assign an implicit contour surface, defined using a ray-linear function, Then the method varies the outer influential radius of the field functions of blended soft objects through the influential radius of the chosen contour surface, so the contour's shape of the resultant transition of the union can become that of the chosen contour surface. This method allows one to determine the shape of the transition by freely choosing a contour surface. In this paper, variable influential radius method has been successfully applied to soft blend and scale union blend in soft object modeling.

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