Application of Operation Research in Logistics and Warehouse Optimization

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Abstract

Logistics and warehouse problems are ill structured and real-world problems. The main reason behind conducting this research is the high degree of application of operation research required to solve these disordered problems. Operation research has a long tradition in reducing costs and improving solutions. Logistics and warehouse optimization is commonly required in most of the industries like automobile, FMCG, container terminals, defence, etc. This paper will provide an overview of the inbound and outbound transportation in warehouses, problematic issues faced, the mathematical methods used by the firms to minimize the transportation cost, application of queueing theory, as well as the limitations of the current system. The first part of the paper will deal with application of network optimization network and queueing theory to predict the waiting time of the trucks for loading/unloading purposes whereas, the second part would deal with problems like selecting the transport route to minimize the time and cost of transportation.

Keywords: Queueing theory, Network optimization problems and simulation.

Introduction

Operation Research incorporates a wide range of mathematical problem-solving techniques and methods, applied in the pursuit of better decision making and efficiency, such as simulation, mathematical optimization and other. There is a sizeable distance between the manufacturing location and the demand areas due to availability of factors of production such as raw material, labour, water. So the good from various locations are collected and warehoused or assembled till the final product for the consumer is ready.

Warehouse optimization is essential to transfer the material from the manufacturer to end user in the shortest possible way, in shortest possible time in requested quality and quantity at minimum costs. Thus, Operations research methods can help identify and help reduce the costs and best line of implementing supply chains. In warehouse optimization, not only the process related to income and expenses established with the material are subjected, but also process of packaging, sorting, labelling and many others.

Operations Research is also used to determine the density of terminal networks at warehouses, size and capacity of warehouses, and others.

Network Optimization Problems

Network optimization is a model in operations research which works like a special type of linear programming model. Networks are an imperative subclass of linear programs that are intuitive, easy to solve, and have good integrity properties. Networks provide a useful way to solve problems even if there are additional constraints or variables that prevent use of networks for modelling the entire problem.

However, Network models have three main advantages in general:

1. They can be solved faster than linear programming problems. Using Network Optimization, problems whose linear program would have 1000 rows and 30,000 columns can be solved in a matter of seconds. This allows network models to be implemented in many applications (such as real-time decision making) for which linear programming would be unsuitable.

2. They have naturally integer solutions. By identifying that a problem can be framed as a network program, it is possible to solve special types of integer programs without resorting to the unproductive and time-consuming integer programming algorithms.

3. They are intuitive. Network models provide a language for talking about problems that is much more instinctive than the variables, objective, and constraints used in the language of linear and integer programming.

There are drawbacks of network models as well such as the fact that they cannot formulate the wide range of models that linear and integer programs can. However, they occur often enough that they form a significant tool for real decision making.

A part of Network Optimization problems is the Minimum Cost Flow Method which is an optimization and decision problem solving

methodology used to find the cheapest possible way of sending a certain amount of flow through a flow network. An example of a flow network is given later. A typical application of this problem involves finding the best delivery route from a factory to a warehouse where the road network has some capacity and cost associated. The minimum cost flow method is one of the most fundamental among all flow and circulation problem solving methods because most other problems can be cast as a minimum cost flow problem and also that it can also be solved efficiently using the network simplex algorithm. Another method of solving minimum cost flow problems can be solved by linear programming, since we optimize a linear function, and all constraints are linear.

A characteristic Logistics Network consists of the following:

1. Facilities: - Plants/Vendors Ports Warehouse Retailers/Distribution Centers Customers
2. Raw materials and finished products that flow between the facilities.

Logistics Network Optimization

The field of Operations research plays a significant role in Logistics Network Optimization. The optimization model which aims to obtain the desired results based on mathematical inputs and assumptions is derived from the precise mathematical procedures gained from operations research, offer the best or optimum solution based on a given implemented formula. This model is based on mathematical and statistical models only. A strategic supply chain network with the perceptibility, competence and flexibility to execute more competitively results in a number of efficiencies. The primary driver for assessing a logistic network design is sometimes an effort to reduce cost, or an effort to improve service, and often, or both. The right network design optimizes transportation, distribution operations, and inventory investment to strike the ideal balance of cost and service.

Some benefits of Logistics Network Optimization are given as follows:

- Reduced logistics costs
- Improved distribution network and asset utilization
- Increased inventory turns
- Improved customer delivery compliance
- Increased carrier capacity and collaboration
- Improved cost to service
- Reduced cross functional waste
- Increased visibility to out of plan/network activity to promote root cause awareness and corrective action
- Supply chain resiliency – ability to survive, adapt, and grow during turbulent change

Let us try to understand this by going through the process of designing a Logistics Network for cost and time optimization:

Strategic Decisions to be taken while designing a typical Logistics Network, to make such decisions companies take the help of mathematical and statistical model.

From the perspective of warehousing, one has to consider constraints and benefits in order to truly come up with an optimized logistics network. Assuming that plants and retailer locations are fixed, we concentrate on the following strategic decisions in terms of warehouses.

1. Deciding the optimal number, location, and size of warehouses,
2. Determining optimum sourcing strategy; which plant/vendor should produce which product,
3. Determining best distribution channels; which warehouses should service which retailers etc.?

The objective is to design or reconfigure the logistics network so as to minimize annual system-wide costs, including production/ purchasing costs, Inventory carrying costs, facility costs (handling and fixed costs) and Transportation costs.

That is, we would like to find a minimal-annual-cost configuration of the distribution network that satisfies product demands at specified customer service levels.

An Example of a typical trade-off involved in this decision-making process:

http://mat.gsia.cmu.edu/classes/QUANT/NOTES/chap11.pdf
Balancing the costs of opening new warehouses with the benefits of being near the customer, quicker supply advantage and healthier inventory handling advantages that are availed by increasing the number of warehouses.

This constraint is explained graphically in figure number 2: The optimal number of warehouses is at the lowest point in the total cost curve which represents the maximum number of warehouses that can be allocated at the lowest costs.

Optimization of Warehouse processes

Once the goods are produced, the manufacturing unit sends the goods to the warehouse for the storage as well as packaging purposes. There are various operation research theories such as queueing theory, inventory model, etc. which are used in optimizing the warehouse processes. The typical operations performed by warehouses are as under:

- Receiving
- Put Away
- Order Picking
- Storing
- Packaging
- Dispatching

Let’s see how the Queueing theory [3] of operation research is applied to maximize the working capacity at a given cost or minimize the warehousing cost. Queueing theory is the mathematical study of waiting lines or queues. This theory is basically used to predict waiting time based on queues and to calculate the resources required to minimize the waiting time. When the requirements enter the system, it is then serviced by service lines or servers and if the lines are busy, then the requirement has to enter into the queue and will be serviced in the order of the queue.

In case of warehousing, the resources required would be labour, conveyor belt points, offloading bays, e-points, etc. The requirements that would have to wait in queue would be trucks at the gates and goods at various service points in the warehouse.

The three important components of queueing theory are:
- Arrival process
- Service Mechanism
- Queue characteristics

**Arrival Process**: Arrival process is how the requirements arrive at the operation place, i.e., singly or in groups. It is also concerned with the time intervals at which the requirements reach the operation place.

**Service Mechanism**: Service mechanism involves the service time required to perform particular task by the server on per unit basis. It also covers information about the resources required to initiate the service, the No. of service lines available, whether the lines are parallel or in series.

**Queue characteristics**: This component determines the way in which the customers or requirements waiting in the line will be choose to serve next. There are following disciplines to choose the requirement:
- First In First Out- Under this method, the customer with the longest waiting will be served first.
- Last In First Out- Under this method, the customer with the shortest waiting will be served first.
- Priority- The customers with high priority are served at first.

In case of warehousing, the operation starts from the point of arrival of trucks loaded with goods. The arrival process is continuous in nature as the trucks keeps on coming in the warehouses. The waiting time of the trucks depends upon the number of offloading bays the warehouse contains and the time required by bays in offloading each truck. The queuing characteristics generally followed by warehouses is FIFO which the next truck with the longest waiting time will be served first. Around 10% \(^4\) of the warehousing costs are incurred at receiving stage.

Service lines in warehouse includes the offloading bays where the goods are offloaded, the E-points where the details of the goods like weight, category, and other characteristics are recorded. After the details are recorded, the goods are put away by putting it on the conveyor belts which directs the goods to the storage area. The goods are then again serviced by the labours at the storage area as they to classify the goods on the basis order frequency before stacking. The goods with maximum order frequency, i.e., utmost priority are put near to the outgoing gates in order to minimize the waiting time for loading. Therefore, in warehousing, the service lines are in series as next operation takes place at the end of previous operation.

When the goods are in demand, the pick-up list is given to the employees to find the required goods from the pallets and pack the goods. The picking and packaging operations alone costs around 55% of the warehouse costs as it includes small processes like searching, travelling, paperwork, etc. Once the goods are packed, they are put to another service line where their details are recorded again and is sorted for identification by putting barcodes, and RFID tags.

While loading goods, the waiting time for vehicles depends upon the type of equipment used for loading like hydra or is done by labours. Here, the loading of goods done is also the service line of the queueing model. The warehousing operation ends as soon as the goods are dispatched.

Thus, Queueing system is suitable in the detection of information about:

- No. of items in stacks closer to the gates
- Average waiting time over to get serve at each service line
- No. of services lines available are either sufficient or not.
- What would happen if there is breakdown in any of the service lines?

Based on this information, it is possible to choose optimal number of service lines as well as area of warehouse which, in turn, would provide number of workers to be employed.

### Vehicle Routing Problems

Once the goods are dispatched from the warehouse, the transporter or logistics department of company has to select the shortest or the optimum route in order to provide deliver in the shortest possible time and at the minimum cost. In order to select such route, vehicle routing problems are used.

Vehicle routing problems are difficult problems to find the most feasible option, applying in areas like transportation, logistics, communication and manufacturing. The most common problem called the capacitated vehicle routing problem (CVRP) requires finding a solution to a simplified transport problem where a number of identical vehicles are delivering to the customers from central spot, and there is a restriction on the number of customers a vehicle can deliver to. The CVRP is an extension of the traveling salesman problem.

These problems can be distinguished into the following types:

1. Assigning vehicles to customers and making routes when there are multiple depots, a heterogeneous set of vehicles, multiple periods, in which customers have to be served more than once and split deliveries where customer is served one or more vehicles.

2. When the researcher has to make sequences, including backhauls and pickup-and-delivery where deliveries are made customers and goods are picked up from particular spots, and orders can be received dynamically, depending on the constraints such that goods should be delivered before others, and also where there are multiple trips, in which vehicles can depart from and return to the central spot more than once.

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3. Problems where there are fixed sequences, like time frames in which each customer has to get the goods or networks with time dependent features such as costs or travelling time.

VRP has many solution methods which are exact and heuristic in nature, even after methods are tailored for solving a specific problem, exact methods for VRP can only solve problems with a constraint having customers up to 200 only.

There are three main different approaches to modelling the VRP, [6]

1. **Vehicle flow formulations**- Using integer variables, the solution is found. This is for basic VRP’s and it’s suitable for problems where cost can be expressed as the sum of costs related to deliveries. This method cannot handle many practical applications.

2. **Commodity flow formulations**- Additional integer variables are used representing the flow of commodities along the paths travelled by the vehicles. This has only recently been used to find an exact solution.

3. **Set partitioning problem**- These have an exponential number of binary variables which are each associated with a different feasible circuit. The VRP is then instead formulated as a set partitioning problem which asks what is the collection of circuits with minimum cost that satisfy the VRP constraints. This allows for very general route costs. This method required exponential number of binary variables which are associated with different feasibility. The problem is adjusted as a set partition problem which is solved for minimum cost satisfying the constraints.

### Role of Simulation in designing Logistics Model

Due to immense competition in today’s business world, when new products with a short life cycle are introduced in the markets, it becomes important for enterprises to invest some attention in their logistics systems. Thus, it is important to see how simulation can be used for designing, analyzing and managing logistics and transportation systems.

### Current Approaches

An L&T (Logistics and Transportation) consists of hubs connected by a set of traffic lanes. Thus the networks form hub-and-spoke arrangements between destinations. L&T systems utilize several resources such as direct resources (used in physical transportation of freight from one destination to another) and indirect resources (used in storing, handling and consolidating at various terminals). However, as time is passing, the conventional L&T systems that guarantee on time saving, less damage, and fast delivery to customers have become extremely complicated and result in increasing costs and decision support systems. Thus descriptive modeling of L&T is becoming more popular in many companies.

L&T problems appropriate for simulation can be categorised as New Design (comprising of network designing, least cost transportation modes and route planning), Evaluation of Alternatives Design (comprising of transportation mode alternatives and service performance) and Refinement and Redesign of Existing Operations (for operational performance analysis)

Simulation models in L&T domains are built for strategic planning, tactical planning, traffic control and dispatching.

The major L&T problem domains are Distribution and warehousing, Trucking Operations, Truck docks and Ramp operations in air cargo hub, the major and most frequently arising problem being Distribution and warehousing.

Previously, distribution has been regarded as a source of cost. Then steps have been taken in order to improve the operating efficiency and cost minimisation. This has led to an improvement in vehicle utilisation and material handling systems. Production shall be planned in such a way that it takes into account marketing, distribution and other activities. Logistics represents an action to bring distribution and other related tasks to the main stream of planning. Thus, planning effective distribution strategies has become an important factor for growing companies.

### Logistics and Distribution

Logistics is the most important factor that affects decisions regarding distribution. The integration of transport, warehousing and inventory management under the banner of distribution management is a frequent consequence of taking a logistics-based approach. Logistics implies the need for vertical integration with supply chain resulting in marketing regarding distribution. This implies a completely different distribution system. [6]

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[6] [http://www.uh.edu/~lcr3600/simulation/app-lt.html](http://www.uh.edu/~lcr3600/simulation/app-lt.html)
Several changes in technology and demand for products have also lead to markets being very volatile. This has led to problems for logistics management. This has created substantial problems for logistics management. Short life periods means shorter lead times, which is defined as the elapsed period of time from receipt of customer order to delivery. Also because production lead times are shortening, through the use of new technology, this in effect means transport and storage account for a greater proportion of the total procurement-to-deliver lead-time. Thus skilful integration of logistics and distribution is extremely important to maximise value through customer service.

Data Requirements for a Warehouse Simulation Model

The input data required is:
- Number of plants
- Number and locations of warehouses
- Number of customers
- Customer demand to warehouses
- Part numbers produced at different plants
- Bill of materials
- Transportation times

Thus most of this data vary over time, hence correspond to some probability distribution generated over time by information collected over a period.

Simulation Outputs and Responses

The list of responses or outputs that a logistics user is interested in knowing about is:
- **Average utilization**
  - Warehouse
  - Trucks
  - Airplanes
- **Inventory levels**
  - Production plants
  - Warehouses
- **Transportation delays**
  - Between plant and warehouse
  - Between warehouse and customers
- **Customer orders**
  - Average waiting times at a warehouse
  - Number waiting at a warehouse

As industries are growing, the need for freight transportation will continue to increase and the L&T systems will become more complex. In order to build a strong system of distribution and transportation, L&T companies will have to invest their time and money for a manageable, cost effective, scientific and clear strategy. This means that applications of mathematical modelling and numerical solution techniques such as simulation will continue to grow in L&T companies.

**Conclusion**

In the process of conducting the research our group visited some of India’s leading logistics companies. It is important to note that these companies have an enormous amount of data to organise and have to execute several managerial decisions in order to maximise productivity and minimize costs. Both of these processes incorporate tools of Operational Research to a heavy extent. Operations research makes use of a number of scientific methods combined with reasonable planning to solve real life logistics and warehousing problems.

As discussed in this project Logistics management includes planning, organization, coordination and managerial decision making all of which can be achieved using mathematical models that we have discussed. For example: A Logistics company will need to choose from various routes to conduct the delivery of a product from its manufacturing stage right to the hands of the customer; minimization of costs and execution under the time constraint becomes of paramount importance. Functions studied under Operations research such as network designing not only help in these designing alternatives but also provide a simple yet effective solution to the company which can be implemented on the ground level.

Through this project we saw numerous applications of models from the discipline of operations research and how they can be implemented effectively to optimally solve logistics and warehousing problems. We started with Network Optimization Problems which provide a simpler way of solving real time problems using variables and constraints used in the language of linear and integer programming. To elucidate we discussed Minimum Cost Flow Method which is an optimization and decision problem solving methodology used to find the cheapest possible way of sending a certain amount of flow through a flow network.

We moved on to comprehensively summarize application of the Queuing theory in the process of warehousing process optimization. We discussed the Arrival process, Service Mechanism and Queue Characteristic as components of the Queuing theory and studied how various disciplines and decisions impact our decision-making process. Then we used vehicle routing problems to show how the optimum transport can be selected. It was followed by the simulation method with which one easily design the whole logistics system.

In order to support India’s fast-paced economy, growth of logistics industry is very essential. It is estimated that...
the Indian logistics industry will continue to show robust growth of 10-15% annually, leading the pace of growth of the economy at large.

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