

CONCEPT OF POINT-TO-POINT WIRELESS NETWORKS VULNERABILITY BASED ON COMPONENT ORDER EDGE CONNECTIVITY

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Abstract

The vulnerability parameter is one of important key issues in assessing the performance of point-to-point wireless networks. One counter-measure to address this vulnerability of wireless network is edge-connectivity combined with the minimum cycle mean algorithm. In an existing method, Chartrand method, proved that if the minimum degree of a network is the floor of half the number of nodes representing stations, then its edge-connectivity equals its minimum degree. A more selective concept of edge-connectivity is introduced, called component order edge connectivity as a new vulnerability parameter that concerned with how this parameter to classical edge-connectivity interrelates. The minimum cycle mean algorithm was also combined together with the edge-connectivity concept. Therefore, we propose a new point-to-point wireless network topology in ending up more invulnerable network, called Fan Networks Model.

Introduction

In graph theories and telecommunication networks, graph connectivity models of wireless networks refer to point-to-point connection of nodes and edges that are designed to represent wireless network topologies. The related topic consists of measuring the degree of a network's vulnerability in particular channel types of connection situation in wireless networks.

To obtain such measures, wireless network models by a graph $G(V,E)$ in which the station terminals are represented by the nodes $V(G)$ and the links that are connecting between nodes are represented by the edges $E(G)$. Graph theoretic techniques provide a convenient tool for the investigation of the vulnerability of a communication to damage from natural or enemy – induced damage. Thus, a wireless communication network can often be represented as interconnection of stations (transmitter and receiver) and links that connecting between two stations.

The assumption is that the network is subject to natural failure or enemy attack aimed at isolating stations from each other. The principle idea underlying the discussion of this topic is the problem of determining the vulnerability and designing wireless network which are invulnerability to en-

emy attack are of paramount importance. As yet, neither the analysis nor synthesis problems have been completed proved, although a number of practical results have been discovered. One difficulty immediately encountered in vulnerability studies is lack of a completely appropriate vulnerability criterion.

An example of a network conditions that has number of links to each station to avoid the occurrence of a communication link disconnection arise. It was proven that if the minimum degree of a graph is at least the floor of half the number of nodes, then its edge connectivity equals its minimum degree, i.e., the network is invulnerable to failure if fewer than the minimum number of links to any station fail. Furthermore, if there is failure of some links and results in some pair of stations no longer able to communicate, then the network has failed [5].

Since then attention has been given to study the parameters of traditional vulnerability. Another network model in which it is not necessary that the surviving edges form a connected subgraph as long as they form a subgraph with a component of some predetermined order, say k , that can still communicate, regardless of whether the network is connected. Thus, they introduced a new edge – failure model as a new vulnerability parameter, namely which is the minimum number of links that failed to produce a network in which the number of stations in each collection of stations that still communicates is less than a predetermined number k . This parameter depends on the value of k , the number of stations needed to communicate. Let $2 \leq k \leq n$, where n is the number of stations connected in the network. As k increases, the value of the parameter decreases, i.e., fewer links need fail in order for the network to fail [3].

In the case that we need all stations to communicate, this new parameter is equal to the traditional parameter. In addition, the failure of all the links to one station results in a network which has failed only when we needed all stations to communicate. Thus, it may be possible that the value of the new parameter may be greater than the minimum number of links to any station. The discussion in this paper, the writers focuses on introducing the concept of network vulnerability in practice that relates in point-to-point wireless networks, for instance, terrestrial point-to-point wireless cellular network and wireless microwave network.

Channel Model in Wireless Network

A wireless network can be seen as a collection of channels sharing space and some common frequency band. Each channel consists of a set of transmitters trying to send data of information to a set of receivers. The simplest channel is point-to-point channel which involves a single transmitter directing at sending data to single receiver and it comes in, for example, the terrestrial wireless networks including cellular wireless networks and wireless microwave networks.

The broadcast channel is one-to-many different data of information to different receivers and it arises in, for example, the downlink of a GSM/HSPA Networks. The multiple access channels is the converse, with several transmitters directing at sending different data of information to a single receiver. This many-to-one situation arises in, for example, the uplink of GSM/HSPA Networks. These channels are also referred to a network links, many of which will be bidirectional active any given time.

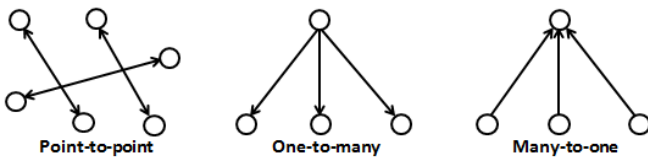


Figure 1. Three channel types of connection in wireless networks

The Wireless Networks Model

Wireless networks are represented by graphs whose nodes correspond to the stations (transmitter-receiver), and edges correspond to links. Therefore, a graph $G = (V,E)$ or G represent a wireless network that consists of a non – empty set of nodes V and a set E of two element subsets of V . If $\{u,v\} \in E$, we say that $\{u,v\}$ is incident at the nodes u and v are adjacent. If $|V| = n$ nodes, and $|E| = e$ edges, G is referred to as an (n,e) graph; n is the order of G and e is the size of G .

In traditional edge-failure model [4,5], it is assumed that nodes are perfectly reliable but edges can fail. When a set F of edge fail, we refer to F as an edge-failure set and the surviving subgraph $G - F$ as an edge-failure state if $G - F$ is disconnected.

Definition 1 [5]:

The edge connectivity of G , denoted by $\lambda(G)$ or simply λ , is defined to be $\lambda(G) = \min \{ |F| : F \in E, F \text{ is an edge failure set} \}$

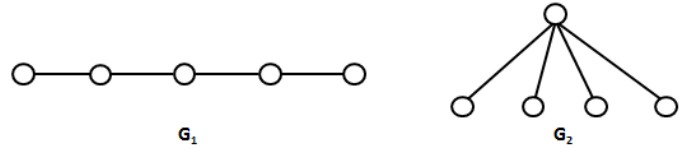


Figure 2. Graphs G_1 and G_2 with

$$\lambda(G_1) = \lambda(G_2) = \delta(G_1) = \delta(G_2) < \left\lfloor \frac{n}{2} \right\rfloor$$

The graphs G_1 and G_2 are represented point-to-point wireless networks of terrestrial wireless cellular and microwave networks, respectively. Both graphs have the same

$$\delta(G) = 1 < \left\lfloor \frac{n}{2} \right\rfloor \text{ and } \lambda(G) = 1.$$

One drawback of the traditional edge-failure model is that the graph $G - F$ is an edge-failure state if it is disconnected, and no consideration given whether or not there exists a relatively large component which in itself may be viable. Therefore, Boesch et al. [3] introduced a new edge-failure model, called component order edge connectivity. In this model, when a set of edges F fail, we refer to F as a k -component edge-failure set and the surviving subgraph $G - F$ as a k -component edge-failure state if $G - F$ contains no component of order no more than $k - 1$, where k is a predetermined threshold value.

Definition 2 [1,3]. Let $2 \leq k \leq n$ be a predetermined threshold value. The k -component order edge-connectivity or component order edge connectivity of G , denoted by $\lambda_c^{(k)}(G)$ or simply $\lambda_c^{(k)}$, is defined to be $\lambda_c^{(k)}(G) = \min \{ |F| : F \subseteq E, F \text{ is } k\text{-component edge-failure set} \}$, i.e., all component of $G - F$ have edge-failure set, i.e., all component of $G - F$ have order $\leq k - 1$.

Definition 3 [1,3]. A set of edges F of graph G is $\lambda_c^{(k)}$ -edge set if and only if it is a k -component order edge-failure set and $|F| = \lambda_c^{(k)}$.

Next we compute $\lambda_c^{(k)}(G)$ for two examples of the graphs of Figure 2, known as the path network, $P_n(G)$, and the star network, $K_{1,n-1}(G)$, respectively.

The type of graph we consider is the path on n nodes, $P_n(G)$. Starting at a pendant edge label the edges consecutively from 1 to $n - 1$. Let F be the set of edges whose label is divisible by $k - 1$. The deletion of the edges in F creates $|F| + 1$ components, the first $|F|$ having order exactly $k - 1$ and the last order at most $k - 1$ (but at least 1). We also note that if fewer edges are removed then will be fewer components and thus by the Pigeonhole principle, there must be

one component of order at least k . Since $|F| = \left\lfloor \frac{n-1}{k-1} \right\rfloor$ we have:

Theorem 1 [1,3]. Given $2 \leq k \leq n, \lambda_c^{(k)}(P_n) = \left\lfloor \frac{n-1}{k-1} \right\rfloor$.

The second type of graph we consider is the star, $K_{1,n-1}$. Deletion of any set of m edges results in a subgraph consisting of $m+1$ components, one isomorphic to $K_{1,n-m-1}$ and the remaining components isolated nodes. Therefore a k -component edge-failure state exists if the component $K_{1,n-m-1}$ contains at most $k-1$ nodes. Thus $n-m \leq k-1$ or $n-k+1 \leq m$. Since component order edge connectivity is the minimum number of edges whose removal results in a k -component edge-failure state, we obtain the following result:

Theorem 2 [1,3]. Given $2 \leq k \leq n, \lambda_c^{(k)}(K_{1,n-1}) = n-k+1$.

From above computations of vulnerability network, for any n nodes we conclude that for $k \geq 3$, point-to-point terrestrial cellular networks are more invulnerable network comparing than terrestrial microwave networks, or give

$$\lambda_c^{(k)}(P_n) < \lambda_c^{(k)}(K_{1,n-1})$$

For $k = 2, \lambda_c^{(k)}(P_n)$ and $\lambda_c^{(k)}(K_{1,n-1})$ that represented point-to-point terrestrial networks, respectively, show that

$$\lambda_c^{(2)}(P_n) = \lambda_c^{(2)}(K_{1,n-1})$$

Definition 4 [2]. The minimum cycle mean (MCM) decreases the complexity of deciding the iteration bound to the problem of finding the MCM of a graph.

The algorithm occupies the concept of a cycle mean, the maximum cycle mean, and the MCM. The cycle means of a new graph G_U are used to compute the iteration bound, and G_U can be seen from DFG for which we will compute the iteration bound (called as DFG G). The cycle mean of a cycle c in G_U is

$$\frac{\text{max comp. time of all cycles in } G \text{ that contain the delays}}{\text{the number of delays in these cycles in } G}$$

To compute the maximum cycle mean of G_U , the graph \bar{G}_U is constructed from G_U by simply multiplying the weights of the edges by -1 as shown as in Fig. 3.

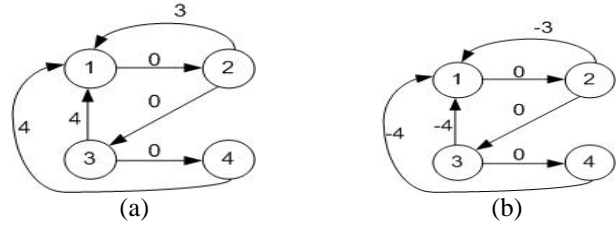


Figure 3. The graphs (a) G_U and (b) \bar{G}_U for the DFG

Fan Networks Model

In order to build a more reliable network, we can utilize both configurations as a new one network. In [1], under the supervisory of Boesch et al., proposed fan network model that combines path and star networks, as depicted in Figure 4 below.

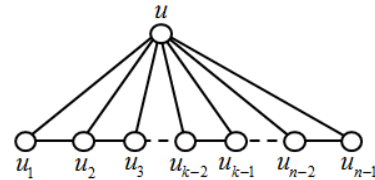


Figure 4. A fan graph, F_n .

Definition 5 [1]. The fan of order n, F_n , is the graph formed by connecting a single vertex to all the vertices of a P_{n-1} .

We give the result of $\lambda_c^{(k)}$ that is stated without proof.

Theorem 3 [1]. Given $2 \leq k \leq n$, then

$$\lambda_c^{(k)}(F_n) = \begin{cases} (n-k+1) + \left\lfloor \frac{n-2}{k-1} \right\rfloor, & \text{if } (k-1) \text{ does not divide } (n-2) \\ (n-k+2) + \left\lfloor \frac{n-2}{k-1} \right\rfloor, & \text{if } (k-1) \text{ divides } (n-2) \end{cases}$$

This fan networks model shall follow the definition described in the **Definition 4**. Hence the proposed fan network model will have optimized minimum value between two consecutive points in wireless networks. The graph between two vertices is a model of a channel between two points.

Analysis and Conclusion

The classical edge-connectivity discussed in [5] proved that if $\delta(G) < \left\lfloor \frac{n}{2} \right\rfloor$, where $\delta(G)$ is the minimum degree of any node in the connected network G of order n , then the edge-connectivity $\lambda(G) = \delta(G)$. We proceed that the simi-

lar result uses of $\lambda_c^{(k)}(G)$, which is the minimum number of edges that must be removed to disconnect the network into components each of order less than k . The investigation gave that for all connected network G of order n there exists a value of such k such that if $\delta(G)$ is at least the floor of half the number of stations, then $\lambda_c^{(k)}(G) \geq \delta(G)$ [4].

We also give an alternative of new point-to-point wireless networks topology, called fan network, and end-up with the result of the network becomes more invulnerable. This gives comparison results representing of fan, star, and path networks, respectively, for $k > 2$. And the measuring performance in the new vulnerability parameter of component order

$$\lambda_c^{(k)}(F_n) > \lambda_c^{(k)}(K_{1,n-1}) \geq \lambda_c^{(k)}(P_n)$$

Acknowledgments

The authors are very thankful to the Institute of Research and Community Empowerment Services -President University for supporting the authors financially. In addition, we would like to express our gratitude to Charles Suffel, Daniel Gross, and L. William Kazmierczak for supporting us with beneficial mathematical knowledge in defining the concept of Component Order Edge Connectivity.

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