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A Computational Method using Analytic Hierarchy Process for Solutions to Bi-Level Quadratic Fractional Programming Problems

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Abstract

Bi-level programming is characterized by two optimization problems located at different levels, in which the constraint region of the upper level problem is implicitly determined by the lower level problem. In this paper, a powerful and robust method which is based on analytic hierarchy process (AHP) is proposed to solve bi-level quadratic fractional programming problems (BLQFPP).We convert thehierarchical system into scalar optimization problem by finding proper weights using AHP so that objective functions of both levels can be combined into one objective function. Since, the objectives of upper and lower level decision makers are potentially conflicting in nature, a possible relaxation of each level decisions are considered by providing weights to objective functions for avoiding decision deadlock. The procedure is not excessively interactive, which most DMs prefer. Theoretical results are illustrated with the help of a numerical example.

Keywords: Bi-level Programming Problem, Analytic Hierarchy Process, Quadratic Fractional Objective, Optimal solution, Satisfactory solution.

1. Introduction

Bi-level programming is a powerful technique for solving hierarchical decision-making problems. In this paper we deal with the bi-level quadratic fractional programming problem (BLQFPP) with the essentially co-operative decision makers (DMs) and propose an approachto solve BLQFPP by weighting method using AHP. The idea of using multiple criteria in order to take decisions has led to the development of a more realistic alternative to the traditional approach: *the multi-criteria decision paradigm.* This aims to optimize simultaneously several objective functions, which may be conflicting and may need therefore, to be dealt with together, as a whole. This paradigm has revolutionized the field of decision theory. However, it has not yet made a significant breakthrough in the field of multi-level (or bi-level) fractional programming problems. Although various non-linear optimization tools are available, the efficiency of these techniques depends to a great extent on the nature of the mathematical formulation of the problem. Some of these traditional techniques, which give accurate results are computationally expansive and become inefficient for a large domain.

A bi-level programming problem (BLPP) is formulated for a problem in which two decision makers (DMs) make decisions successively i.e. BLPP is a sequence of two optimization problems in which the constraints region of one is determined by the solution of second. For example, in a decentralized firm, top management, an executive or headquarters makes a decision such as a budget of the firm, and then each division determines a production plan in the full knowledge of the budget. We can find many instances of decision problem, which are formulated as bi-level programming problem, and concerning the above mentioned hierarchical decision problem in decentralized firm, it is natural that the decision makers behave cooperatively rather than non-cooperatively.

Multi-level optimization plays an important role in engineering design, management, and decision making in general. Ultimately, a designer or decision maker needs to make tradeoffs between disparate and conflicting design objectives. The field of multi-level optimization defines the art and science of making such decisions. The prevailing approach for address this decision-making task is to solve an optimization problem, which yields a candidate solution.

A bi-level programming problem (BLPP) is a special case of multi-level programming problem (MLPP). Multi-level programming problem can be defined as a p-person, non-zero sum game with perfect information in which each player moves sequentially from top to bottom. This problem is a nested hierarchical structure. When $p = 2$, we call the system a bi-level programming problem. Hierarchical optimization or multi-level programming techniques are extension of Stackelberg games for solving decentralized planning problem with multiple DMs in a hierarchical organization. The Stackelberg solution has been employed as a solution concept to BLPPs, and a considerable number of algorithms for obtaining the solution have been employed.

When the Stackelberg solution is employed, it is assumed that there is no communication between the two DMs, or they do not make any binding agreement even if there exists such communication. However, the above assumption is not always reasonable when we model decision-making problems in a decentralized firm as a BLPP in which top management is DM1 and an operation division of the firm is DM2 because it is supposed that there exists a cooperative relationship between them.
Consider a computational aspect to the Stackelberg solution we note that the problem for obtaining the Stackelberg solution is non-convex programming problem with special structure. From such difficulties a new solution concept, which is easy to compute and reflects the structure of multi-level (or bi-level) programming problem is expected. The bi-level programming problem has received increasing attention in the literature [2,3,4,6,7,8,9,10,11,12,13,14,15,17,18,19,20,22,23,24,26,27,28,29].The formulation and different version of BLPP are given by Bard [2,3], Candler [9], Bard and Falk[1], Herminia and Carmen[14] and Bialas and Karwan[7,8]. Bialas and Karwan [7] are the pioneers for linear BLPP who presented vertex enumeration method, called Kth-best solution. These were solved by simplex method. To solve the non-linear problem that arises due to the K-T conditions, Bialas and Karwan [7] proposed a parametric complementary pivot (PCP) algorithm which interactively solves a slight perturbation of the system. Ben-Ayed and Blair [6] showed that the PCP may not converge to optimality. Bard and Falk [1] proposed the grid search algorithm. Based on Bard and Falk’s [1] algorithm, Unlu [27] proposed a technique of bi-criteria programming..

In problems with more than one conflicting objective there exists no single optimal solution but a number of solutions all of which are optimal. Such solutions are called Pareto optimal solutions. In the frame work of fuzzy decision, Bellman and Zadeh [5], Sakawa and Nishizaki [22] proposed a linear programming based on interactive fuzzy programming for bi-level linear fractional programming problems. This method is used to derive the satisficing solution for the DM efficiently from a Pareto optimal solution set by updating the reference membership value of the DM. Adopting the same concept Mishra and Ghosh [19] presented interactive fuzzy programming approach to bi-level quadratic programming problem. Saraj and Safaei [23] used the global criterion method to solve BLPP by an interval approach.Dey and Pramanik [11] presented Goal Programming Approach to Linear Fractional bi-level programming problem based on taylor series approximation. Wanga Guangmin, Wanb Zhongping etl,[28] proposedGenetic algorithm based on simplex method for solving linear-quadratic bi-level programming. Again, Wang Guangmin etl [29] presented Genetic algorithm for solving quadratic bi-level programming problem. Recently, Tao [26] presented A Bi-Level Model to Estimate the US Air Travel Demand.

The estimation of the relative weights of criteria plays an important role in a multiple criteria decision analysis (MCDA) process. Mishra [18]consider the solution of a bi-level linear fractional programming problems (BLLFPP) by Weighting method . A non-dominated solution set is obtained by this method. In this paper we deal with the bi-level quadratic fractional programming problem (BLQFPP) with the essentially cooperative DMs and propose an algorithm to solve BLQFPP by weighting method using AHP[16,18,21,25]. In proposed approach the hierarchical system will be converted into scalar optimization problem (SOP) by finding proper weights for both objective functions using AHP so that objective functions of both levels can be combined into one objective function. Here the relative weights represent the relative importance of the objective functions.AHP is a mathematical technique developed for incorporating multi-criteria decision making and designed to solve its complex problems. AHP and similar methods often use pair wise comparison matrices for determining the scores of alternatives with respect to a given criterion, or determining values of weight vector. It is a simple method to apply to the bi-level systems compared to the other transformation method. The proposed approach really depends on the configuration of the system, it's over all management and the relative importance of a DM with respect to other DMs in the system. Perhaps the most creative task in making a decision is to choose the factors that are important for that decision. AHP can be conducted in three steps: perform pair wise comparisons, assess consistency of pair wise judgments, and compute the relative weights and then, it enables DM to make pair wise comparisons of importance between objectives.

2. Bi-level quadratic fractional programming problem

A bi-level quadratic fractional programming problem is formulated as:

$$\underset{x_1}{Maximise} \quad z_1(x_1, x_2) \text{, where, } x_2 \text{ solves} \qquad (1)$$

$$\underset{x_2}{Maximise} \quad z_2(x_1, x_2) \qquad (2)$$

$$subject \;\; to \quad A_1 x_1 + A_2 x_2 \le b, \quad x_1 \ge 0, x_2 \ge 0. \qquad (3)$$

Where objective functions $z_i(x_1, x_2),\ i = 1,2$ are represented by a quadratic fractional function

$$z_i(x_1,x_2)=\frac{n_i(x_1,x_2)}{d_i(x_1,x_2)}=\frac{x_1{}'Q_{i1}x_1+x_2{}'Q_{i2}x_2+c_{i1}x_1+c_{i2}x_2+c_{i3}}{x_1{}'R_{i1}x_1+x_2{}'R_{i2}x_2+d_{i1}x_1+d_{i2}x_2+d_{i3}} \quad (4)$$

$x_i,\ i=1,2$ are n_i-dimensional decision variables.

Q_{i1} and R_{i1}, $i=1,2$ are $n_1\times n_1$ positive definite matrix.

Q_{i2} and R_{i2}, $i=1,2$ are $n_2\times n_2$ positive definite matrix.

c_{i1} and d_{i1}, $i=1,2$ are n_1-dimensional row vectors.

c_{i2} and d_{i2}, $i=1,2$ are n_2-dimensional row vectors.

c_{i3} and d_{i3}, $i=1,2$ are constants.

b is a $m-$dimensional constant column vector.

$n_i(x_1,x_2)$ is the numerator of the quadratic fractional objective function.

$d_i(x_1,x_2)$ is the denominator of the quadratic fractional objective function.

A_i, $i=1,2$ is a $m\times n_i$ constant matrix; and it is assumed that the denominators are positive i.e. $d_i(x_1,x_2)>0$, $i=1,2$

For the sake of simplicity, we use the following notations:-
$x=(x_1,x_2)\in R^{n_1+n_2}$;

Also, let DM1 denote the DM at the upper level and DM2 denote the DM at the lower level.

In the bi-level quadratic fractional programming problem "(1)", $z_1(x_1,x_2)$ and $z_2(x_1,x_2)$ respectively represent objective functions of DM1 and DM2 and x_1, x_2 represent decision variables of DM1 and DM2 respectively.

3. Generation of non-dominated solution by weighting method using AHP

3.1 Weighting method

Decision-making is the process of selecting a possible course of action from all the available alternatives. Although few optimization tools are available for BLQFPP , the efficiency of these techniques depends to a great extent on the nature of the mathematical formulation of the problem. Some of these traditional techniques, which give accurate results are computationally expansive and become inefficient for a large domain. Weighting method has been widely studied**,** experimented and applied in many fields in engineering worlds. Not only does weighting method provide an alternate method to solving problem, it consistently outperforms other traditional methods in the most of the problem link. Weighting method has no special requirement for the characters and differentiability of the function. Perhaps the most creative task in making a decision with the hierarchical decision making with the hierarchical decision making situations is to choose the factors that are important for that decision. The basic idea of assigning weights to the various objective functions, combining these into a single objective function and parametrically varying the weights to generate the non-dominated set was first proposed by Zadeh in 1963. Mathematically, the weighting method can be stated as follows:

$$\max/\min \quad w_1z_1(\bar{x})+w_2\,z_2(\bar{x})+\ldots\ldots+w_pz_p(\bar{x}) \quad (5)$$

Subject to $\bar{x}\in X$ where X is the feasible region. Thus, a multiple objective problem has been transformed into a single objective optimization problem for which solution methods exist. The coefficient w_p operating on the p^{th} objective function, $z_p(\bar{x})$, is called the weight and can be interpreted as "the relative weight or worth" of that objective function when compared to the other objectives. These weights can be obtained by Analytic Hierarchy Process (AHP).

3.2 Determination of weight using analytic hierarchy process (AHP)

AHP [16,18,21,25,30] is a mathematical technique developed for incorporating multi criteria decision making and designed to solve its complex problems. AHP and similar methods often use pairwise comparison matrices for determining the scores of alternatives with respect to a given criterion, or determining values of a weight vector. AHP can be conducted in three steps:

*perform pairwise comparisons,
* assess consistency of pairwise judgments,
* compute the relative weights and then, it enables DM to make pairwise comparisons of importance between objectives according to the scale[21].

Because human is not always consistent, the theory of AHP does not demand perfect consistency and allows some small inconsistency in judgment and provides a measure of inconsistency. Before computing the weights based on pairwise judgments, the degree of inconsistency is measured by the Consistency Index (CI)[16,21,25]. Perfect consistency implies a value of zero for CI.

If the weights of the various objectives are interpreted as the representing the relative preference of some DM, then the solution to (5) is equivalent to the best compromise solution, i.e., the optimal solution relative to a particular preference structure. Moreover, the optimal solution to (5) is a non-dominated solution provided all the weights are positive. Allowing negative weights would be equivalent to transforming the maximizing problem to a minimizing one, for which a different set of non-dominated solutions will exist. The trivial case where all the weights are zero will simply identify $\bar{x} \in X$ as an optimal solution and will not distinguished between dominated and non-dominated solutions [10].

Thomas Saaty [25], developed the AHP, it becomes a useful tool for estimating judgment elements by quantifying subjective decisions. It is used to derive relative weights of decision elements and then synthesize them to obtain the corresponding weights for alternatives and criteria. It is a building block for decision making. There are many papers applied AHP to solve decision problem. For example, Zanakis et al. [30] studied over 100 applications of AHP in the service and government sectors. On the other hand, other researchers provided different approach to enhance the theoretical background of AHP to refine its analytical derivation. One open question in AHP is how to derive the relative weights from a comparison matrix. The majority practitioner agreed to use the eigenvector method proposed by Saaty [25].

3.3 Method for generating non-dominated solutions

The concept of non-dominated solution was introduced by *Pareto*, an economist in 1896. A preferred (best) solution is a non-dominated solution which is chosen by the DM his self that is lies in the region of acceptance of all DMs. Non-dominated solution is to design the best alternative by considering the various interactions within the design constraints that best satisfy the DM by way of obtaining some acceptable levels of quantifiable objective functions.This method be distinguished with; a set of quantifiable objective functions, a set of well defined constraints and a process of obtaining some trade-off information, between the stated quantifiable objective functions. The most common strategy for finding non-dominated solutions of MLP problems is to convert it into a SOP. DMs provide their preference and converting MLP problem into a SOP by finding vector of weights for objectives.

A non-dominated solution is one in which no one objective function can be improved without a simultaneous detriment to at least one of the other objectives of the vector maximum problem [VMP]. A given multiple objective mathematical problem which contains only maximization type objective functions is called the VMP. A feasible solution $\bar{x}^* \in X$ (decision space) is a non-dominated solution to the VMP iff there does not exist any other feasible solution $\bar{x} \in X$ such that $z_p(\bar{x}^*) \le z_p(\bar{x}),\ p = 1,2$ and $z_p(\bar{x}^*) < z_p(\bar{x}),$ for at least one p.

A non-dominated solution is one for which there is no other solution giving equal or greater values of each and every objective function. But in even the smallest problem, the number of non-dominated solutions generated may be infinite. This is because all points on the line joining two non-dominated and extreme points are themselves non-dominated. A preferred solution is a non-dominated solution which is chosen by the DM, through some additional criteria, as the final decision. As such it lies in the region of acceptance of all the criteria values for the problem. A preferred solution is also known as the 'best' solution.

The most common strategy for finding non-dominated solutions of a multi-level problem is to convert it into a scalar optimization problem (SOP). This class of method does not require any assumption or information regarding the DMs utility function.

4.Solution of bi-level quadraticfractional programming problem using AHP

Let a quadratic fractionalBLPP be represented as:

$$\max_{x_1} \quad z_1(\bar{x})$$
$$\max_{x_2} \quad z_2(\bar{x})$$
$$subject \quad to$$
$$x \in S = \{A_1 x_1 + A_2 x_2 \ (\ge, =, \le) B, x \ge 0\}$$
$$\bar{x}_1 \ge 0, \bar{x}_2 \ge 0, \ldots\ x_p(\bar{x}).$$

Where, $z_1(\bar{x}),\ z_2(\bar{x})$ and $A_i x_i (\ge, =, \le) B$, are quadratic fractional objective functions and linear or non linear constraints respectively. $\bar{x}_1 \ge 0, \bar{x}_2 \ge 0,$ are decision vectors under the control of the upper level and lower level DM.

Let X = set of feasible solutions = $\{\bar{x} : \bar{x} \in R^n,$ $A_i x_i (\ge, =, \le) B\}$,

$\bar{x}$ = decision vector in n-dimensional Euclidean space = $x = x_1 \cup x_2$.

In the weighting problem $\bar{p}(w)$ in the absence explicit preference structure, the strategy is to generate all or representative subsets of non-dominated solutions from which a DM can select the suitable solution.

Solving the SOP involves finding $\bar{x}^* \in X$ such that $z(\bar{x}^*) \geq z(\bar{x}) \quad \forall \quad \bar{x} \in X$. The point $\bar{x}^*$ is said to be global optimum. If strict inequality holds for the objective functions, then $\bar{x}^*$ is the unique global optimum. If the inequality holds for some neighborhood of $\bar{x}^*$, then $\bar{x}^*$ is a local or relative optimum while it is strict local optimum if strict inequality holds in a neighborhood of $\bar{x}^*$.

By using the AHP pair wise comparison process, weights or priorities are derived from a set of judgments. While it is difficult to justify weights that are arbitrarily assigned, it is relatively easy to justify judgments and the basis (hard data, knowledge, experience) for the judgments. suppose already the relative weights of 2-objective functions are known, for 2-level hierarchical objective functions a complete pair wise comparison matrix *A* can be expressed as; $A = [a_{ij}]$, $i, j = 1,2$ is a matrix of size 2×2 with the following properties; $a_{ij} > 0, a_{ii} = 1,$ and $a_{ij} = 1/a_{ij}, i, j = 1,2$ where a_{ij} is the numerical answer given by the each DM for the question "How many times objective *i* is more important than objective?

$$\begin{array}{cc} & \begin{array}{cc} z_1 & z_2 \end{array} \\ A = \begin{array}{c} z_1 \\ z_2 \end{array} & \begin{bmatrix} 1 & a_{12} \\ a_{21} & 1 \end{bmatrix} \end{array}$$

After the normalized matrix, *N* of pairwise comparison matrix *A* for a hierarchical 2-level structure is designed, the normalized principal priority vector can be obtained by some ways such as averaging across the rows where, it shows the relative weights for objectives. The weighting problem is to find the weight vector $W = (w_1, w_2)$ such that the appropriate ratios of the components of *W* reflect or, at least, approximate all the a_{ij} values $(i, j = 1,2)$ given by DMs.

Then, the weighting problem for BLQFPP problem is formulated as follows:

$$P(\bar{w}) = \max_{\bar{x} \in X} \sum_{n=1}^{2} \bar{w}_P z_P(\bar{x}) \qquad (9)$$

subject to

$$g_i(x) \leq b_i, \; i = 1,2,...,m \quad (10)$$

$$\bar{L}_1 \leq \bar{x}_1 \leq \bar{U}_1 \qquad (11)$$

$$\bar{L}_2 \leq \bar{x}_2 \leq \bar{U}_2 \qquad (12)$$

Where, $\bar{w} \in W = \{ \; \bar{w} : \bar{w} \in R^P, \; w_n \geq 0, \; P = 1,2$ and $\sum_{n=1}^{2} \bar{w}_P = 1 \; \}$; and

$$z_i(x_1, x_2) = \frac{n_i(x_1, x_2)}{d_i(x_1, x_2)} = \frac{x_1{'} Q_{i1} x_1 + x_2{'} Q_{i2} x_2 + c_{i1} x_1 + c_{i2} x_2 + c_{i3}}{x_1{'} R_{i1} x_1 + x_2{'} R_{i2} x_2 + d_{i1} x_1 + d_{i2} x_2 + d_{i3}}$$

$\bar{U}_P$ and $\bar{L}_P$ are the upper and lower bounds of decision vector provided by the respective DM. Finally the quadratic fractional programming problem *(9) - (12)*, with a single objective function is solved. Here the weighting coefficients convey the importance attached to an objective function. Suppose that the relative importance of the both objective functions is known and is constant. Then the preferred solution is obtained by solving $p(\bar{w})$ where $\bar{w}_P{'s} \geq 0$ the weighting coefficients are. The $\bar{w}_P{'s}$ are normalized since, $\sum_{n=1}^{2} \bar{w}_P = 1$.

This method can be used to generate non-dominated solutions by utilizing various values of $\bar{w}$. In such a case the weighting coefficients $\bar{w}$ do not reflect the relative importance of the objective functions in the proportional sense, but are only parameters varied to locate the non-dominated points.

5. Numerical example

Consider the following bi-level quadratic fractional programming problem (BLQFPP):

$$\max_{x_1} \quad z_1 = \frac{10x_1^{\,2} + 15x_2^{\,2} + 5}{x_1^{\,2} + 2x_2^{\,2} + 1}$$

$$\max_{x_2} \quad z_2 = \frac{25x_1^{\,2} + 9x_2^{\,2}}{2x_1^{\,2} + x_2^{\,2} + 1}$$

subject to

$$4x_1 - 5x_2 \leq 15; \; 3x_1 - x_2 \leq 21;$$
$$2x_1 + x_2 \leq 27; \; 3x_1 + 4x_2 \leq 45;$$
$$x_1 + 3x_2 \leq 30; \; x_1 \geq 0, x_2 \geq 0.$$

Solution by proposed Solution approach: Let the bounds provided by the respective decision makers be $4 \leq x_1 \leq 20$, $0 \leq x_2 \leq 10$ .The pair wise comparison matrix $\tilde{A}$ of order 2 and its normalized matrix $\tilde{N}$ [18] for the hierarchical objective functions are given as:

$$\tilde{A} = \begin{matrix} & z_1 & z_2 \\ z_1 & 1 & 4 \\ z_2 & 1/4 & 1 \end{matrix};$$

$$\tilde{N} = \begin{bmatrix} 1/1.25 & 4/5 \\ 0.25/1.25 & 1/5 \end{bmatrix} = \begin{bmatrix} 0.80 & 0.80 \\ 0.20 & 0.20 \end{bmatrix}$$

Thus, the normalized relative weights are $\tilde{w}_1 = (0.80 + 0.80)/2 = 0.80$ and $\tilde{w}_2 = (0.20 + 0.20)/2 = 0.20$. Matrix $\tilde{A}$ is consistent (since $\tilde{A}$ is a (2×2) matrix).

The weighting problem is therefore formulated as:

$$P(\overline{w}) = \max_x \quad (\tilde{w}_1 z_1 + \tilde{w}_2 z_2)$$

$$P(\overline{w}) = \max_x \quad \left[\quad 0.80 \frac{10x_1^2 + 15x_2^2 + 5}{x_1^2 + 2x_2^2 + 1} + 0.20 \frac{25x_1^2 + 9x_2^2}{2x_1^2 + x_2^2 + 1} \right]$$

$$subject \quad to$$

$$4x_1 - 5x_2 \le 15;\ 3x_1 - x_2 \le 21;\ 2x_1 + x_2 \le 27;$$
$$3x_1 + 4x_2 \le 45;\ x_1 + 3x_2 \le 30;\ x_1 \ge 0, x_2 \ge 0.$$

The non-dominated solution set is generated by parametrically varying the weights and is tabulated below:

$\tilde{w}_1, \tilde{w}_2$	x_1, x_2	$P(\tilde{w})$
0.0,1.0	5.34,1.27	12.20
0.1,0.9	5.10,1.08	11.94
0.2,0.8	4.90,0.92	11.68
0.3,0.7	4.74,0.79	11.43
0.4,0.6	4.62,0.69	11.18
0.5,0.5	4.51,0.61	10.93
0.6,0.4	4.43,0.54	10.68
0.7,0.3	4.35,0.48	10.44
0.8,0.2	**4.29,0.44**	**10.19**
0.9,0.1	4.24,0.39	9.95
1.0,0.0	4.19,0.36	9.70

Here, we see that even we vary the weight vector, the solution remains more or less the same. This approach determines a subset of the complete set of non-dominated solutions and unique characteristics of a BLPP is reflected by allowing each DM to assign upper and lower bounds for the decision variables under his control.

6. Conclusion

The objective of this paper is to apply weighting approach based on AHP to bi-level quadratic fractional programming problems. The proposed approach is easy to implement. The procedure is not excessively interactive, which most DMs prefer. Proposed approach does not require any assumption or information regarding the DMs utility function. The method has no special requirement for the characters of the function and overcome the difficulty discussing the conditions and the algorithms of the optimal solution with the definition of the differentiability of the function. This method can be used to generate non-dominated solutions by utilizing various values of $\overline{w}$ .AHP gives the relative weights to form a super objective function. We also observe that even though we vary the weight vector, the solution remains more or less the same. Thus the non-dominated solution set reduces to a point. The main advantage of the proposed weighting approach is that the possibility of rejecting the solution again and again by the fuzzy approach and re-evaluation of the problem repeatedly, by redefining the elicited membership functions, needed to reach to the satisfactory decision does not arise. The problem can never be infeasible and unbounded. Proposed method facilitates computation to reduce the complexity in solving problem and is much more effective.

References

[1] Bard J.F. and Falk J.E., An Explicit Solution to the Multilevel Programming Problem, Computers and Operation Research,8(2); (1982): 260-272.

[2] Bard J.F., An Algorithm for Solving the Bi-level Programming Problem, Mathematics of Operation Research 8(2), (1983): 260-270.

[3] Bard J.F. Optimality Conditions for the Bi-level Programming Problem. Naval Research Logistic Quarterly, 31(1), (1984): 13-26.

[4] Benoît Colson, Patrice Marcotte, and Gilles Savard, Bilevel programming: A survey, 4OR, 3(1), (2005): 87–107.

[5] Bellman R.E., Zadeh L.A. Decision Making in Fuzzy Environment. Management Science 17 (1970): 141-164.

[6] Ben-Ayed O. and Blair C.E. Computational difficulties of bi-level linear programming.OperationsResearch, 38, (1990):556-560.

[7] Biales W.F, Karwan M.H. Two-level linear programming, Management

Science,30(80),(1984): 1001-1020.

[8] Bialas W.F, Karwan M.H. On two-level Optimization, IEEE Trans. Automatic Control,AC-27(1982): 211-214.

[9] Candler W and Karwan M.H. A Linear Two Level Programming Problem, Computers and Operation Research,9(1), (1982): 59-76.

[10] Compelo. M and Scheimberg. S, A note on a modified simplex approach for solving bi-level Linearprogramming problems, European Journal of Operational Research,126(1), (2000): 454-458.

[11] DeyPartha Pratim and Pramanik Surapati "Goal Programming Approach to Linear Fractional Bilevel Programming Problem Based on Taylor Series Approximation",International Journal of Pure and Applied Sciences and Technology, International Journal of Pure and Applied Sciences and Technology, 6(2), (2011): 115-123.

[12] Hong Li, Yong-Chang Jiao, etl, An efficient method for linear bilevel programming Problems based on the orthogonal genetic algorithm, International Journal of Innovative Computing, Information and Control,5(9), (September 2009): 2837-2846.

[13] Hecheng Li, Lei Fang, An Evolutionary Algorithm for Solving Bilevel Programming ProblemsUsingDuality Conditions, Mathematical Problems in Engineering, Article ID 471952, (2012)**:** 14 pages.

[14] Herminia I. Calvete, Carmen Galé, Theory and Methodology- The bilevel linear/linear fractional programming problem,European Journal of Operational Research, 114(1), (1999):188-197.

[15] HejaziaS.R., Memariania A. *etl*, Linear bilevel programming solution by genetic algorithm,Computers & Operations Research, 29(1), (2002) :1913–1925.

[16] Jason Chang S.K., Lei Hsiu-li etl., Note on Deriving Weights from Pairwise Comparison Matrices in AHP,Information and Management Sciences,19(3),(2008): pp507-517.

[17] Malhotra Neelam and Arora .S.R.An Algoritham to Solve Linear Fractional Bilevel, OPSEARCH, 37 (1) (2000):1-13.

[18] Mishra Savita, Weighting method for bi-level linear fractional programming problems, European Journal of Operational Research, 183(1), (2007):296-302.

[19] Mishra Savita, Ghosh Ajit. Interactive fuzzy programming approach to bi-level quadratic fractional programming problems, Annals of Operational Research,;143(1), (2006): 249-261.

[20] Mishra Savita, Ghosh Ajit , A Fuzzy Solution Approach to Bi-Level Linear Fractional Programming Problems, International Journal of computer, Mathematical Sciences and applications,1(2-4),(2007):181-189.

[21] Osman M. S. et al, A Compromise Weighted Solution For Multilevel Linear Programming Problems, Int. Journal of Engineering Research and Applications ,ISSN : 2248-9622, Vol. 3, Issue 6, Nov-Dec 2013, pp.927-936.

[22] Sakawa M.. Nishizaki I. Interactive Fuzzy Programming For Two-level Linear Fractional Programming Problems. Fuzzy Sets and Systems;119(31-40);(2001): 81-186.

[23] Saraj M , Safaei N. , Solving Bi-Level Programming Problems on Using Global Criterion Method with an Interval Approach Applied Mathematical Sciences,6(23), (2012): 1135 – 1141.

[24] Thirwani D, Arora S.R. Bi-level linear fractional programming problem. Cahiers Du Cero; 35(1-2),(1983): 135-149.

[25] Satty T. L., The Analytical Herarchy Pocess, Plenum Press, New York, 1980.

[26] Tao Li, A Bi-Level Model to Estimate the US Air Travel Demand, *Asia-Pacific Journal of OperationalResearch.,* (2014): DOI: 10.1142/S0217595915500098.

[27] Unlu G. A. Linear Bi-level Programming Algorithm Based on Bi-criteria Programming, Computers Operation Research, 14(1); (1987): 173-179.

[28] WangaGuangmin, Wanb Zhongping etl, Genetic algorithm based on simplex methodfor solving, linear-quadratic bilevel programming problem,Computers and Mathematicswith Applications 56 (1), (2008): 2550–2555.

[29] Wang Guangmin , Wan Zhongping Wang, Xianjia, Fang Debin, Genetic algorithm for solving quadratic bilevel programming problem**,** Wuhan University Journal of Natural Sciences, 12(3),(May 2007): 421-425.

[30] Zanakis, S.,Mandakovic, T., Gupta S,.Sahay S. and Hong S.,A review of program evaluation and fund allocation methods within the service and government sectors, Socio-Economic Planning Science, Vol.29.No.1, pp.5 9-79,1995.