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The Non-Uniform Scaling Method for Developing Boolean Set Blends with an Individual Blending Range Parameter for Each Primitive to Adjust its Subsequent Blend in Soft Object Modeling

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Abstract

In implicit surface modeling, soft object modeling is getting attraction because it has lower computing complexity of union blends. However, existing set blends for soft objects always behaves like a pure union or intersection, *Max*() or *Min*() function, in non-blending regions for blending range control. As a result, when they are reused as a new primitive in other blends, their primitives always have similar subsequent blending surfaces with primitives in the later blends. To solve the problem, this paper proposes the non-uniform scaling method. This method is a generalized method that can transform an existing union blend to a new union or intersection blend and additionally provide each primitive an individual subsequent blending range parameter for adjusting the blending range and size of each primitive's subsequent blends. Thus, primitives of new blends are allowed to have different subsequent blending surfaces in later blends. Besides, through the proposed method, new two and high dimensional super-ellipsoidal union and intersection blends are also developed for soft object modeling.

1. Introduction

In soft objects modeling, a soft object is represented by a 0.5 level surface of a non-negative and decreasing field function. A complex object is given by performing Boolean set blends on primitive soft objects, such as planes, ellipsoids, skeletons,..., etc. Set blends includes union, intersection and difference and they are applied to join intersecting primitives with transition surfaces additionally and automatically generated to erase and avoid non-smooth and sharp edges, kinks, and creases. Especially, because field functions are decreasing monotonically, a union of soft objects is obtained by soft blends [1, 2], requiring summation only. Existing field functions can be found in [1, 2, 3, 4, 5, 6, 7, 8, 9]. In the literature, existing set blends in soft object modeling include:

(1). Pure union and intersection blends, *Max* and *Min*, were proposed in [10], but they are C^0 continuous only and hence might generate non-smooth surfaces.

(2). Super-ellipsoidal union blend [10] simulates *Max* union. It offers a curvature parameter n for varying the shape of the resulting transition on the blend and has C^n continuity.

(3). A full family of set blends, union, intersection and difference, was proposed in [8], which do sum and product operations only.

(4). To have better shape and size control on the transitions of the resulting blending, blends with blending range parameters and C^1 continuity were developed. Blending range parameters are used to adjust the size of the transitions within a specific region without deforming primitives totally after blending, and C^1 continuity allows them to be used to generate sequential blends. Regarding these, two-dimensional elliptic blends were developed in [11] and high-dimensional super-ellipsoidal blends in [12]. High-order continuous blend was also developed in [13]. On the other hand, gradient-based blending ranges were also proposed in [12, 14, 15] to do bulge elimination.

(5). Field functions offering inner and outer influential radii parameters were proposed in [7, 9]. Adjusting the inner and the outer radii parameters enables soft blend to vary their primitives' blending range in blending.

(6). Blends in [11, 16, 17] enable users to define a free-form blending curve point-by-point so that a free-form transition's profile can be generated

Regarding the scale method [12], it can develop blends $B_k(f_1,...,f_k)$ with blending range parameters $r_1,\ldots, r_k$, which offer a better shape and size control over the transitions and deform primitives locally with blending regions. In fact, local deformation of primitives after blending implies that B_k behave like a pure union $Max(f_1,...,f_k)$ or an intersection $Min(f_1,...,f_k)$ in non-blending regions after blending. Unfortunately, this also incurs the following difficulty in sequential blends. For example, in $B_2(B_k(f_1,...,f_k), f_{k+1})$ with range parameters r_a and r_b for B_k and f_{k+1} in B_2, primitives $f_1,...,$ and f_k always have the same blending range r_a to blend with f_{k+1}, and hence they have similar transitions in blends with f_{k+1}.

Therefore, the paper proposes the non-uniform scaling method. This method is capable of transforming an existing union blending operator with range parameters $r_1,\ldots, r_k$ into a new union or intersection blending operator $B_k(x_1,...,x_k)$, which additionally give each primitive a new subsequent blending range parameter m_i, i=1 to k. As a result, like $B_2(B_k(f_1,...,f_k), f_{k+1})$ stated above, f_1,..., and f_k can have different subsequent blends with f_{k+1}, determined by m_i and r_a. That means the newly proposed blends offer respective blend range adjustments on primitives' subsequent blend's transitions.

Since this non-uniform scaling method is a general method for transforming an existing blend into a new blend, this paper furthermore develops high-dimensional super-ellipsoidal and two-dimensional elliptic union and intersection blends from the proposed method.

The remainder of this paper is organized as follows. Section 2 describes related works and the difficulty in details. Section 3 presents the non-uniform scaling method. Section 4 introduces differential new blends created from the proposed method in Section 3. Conclusion is given in Section 5.

2. Related works of soft objects

This section presents the definition of soft objects first and then describes blends on soft objects and their difficulty.

2.1. Definition of a soft object

A primitive soft object is defined using a field function $f_i(x,y,z)$:$R^3 \rightarrow [0, 1]$, i=1,...,k, and written as the level set

$$\{(x,y,z)\in R^3 \mid f_i(x,y,z)=0.5\},$$

denoted as f_i=0.5 interchangeably. $f_i(x,y,z)$ is usually written as a composition of a potential function P and a distance function d_i by:

$$f_i((x,y,z))=P(\ d_i(x,y,z)\ ).$$

$P(d)$ maps R_+ to [0, 1], $R_+\equiv\{x\in R \mid x\geq 0\}$, and it is decreasing. And $d_i(x,y,z)$ maps R^3 to R_+ and it decides the shape of $f_i(x,y,z)$=0.5. Existing distance functions include sphere, ellipsoids, super-ellipsoids, cylinders, sweep objects,..., etc., in [1, 2, 3, 4, 5, 6, 18, 19, 20]. Existing potential functions are founded in [1, 2, 3, 7, 8, 9].

2.2. Set blends of soft objects

Furthermore, a complex soft object is constructed from primitives f_i=0.5, i=1,...,k, and written using a blending operator $B_k(x_1,...,x_k)$: $R_+^k \rightarrow$ to R_+ by

$$B_k(f_1,...,f_k)=0.5,$$

which is called a blending surface on f_i=0.5, i=1,...,k, too. The role of $B_k(x_1,...,x_k)$ is to connect primitives f_i=0.5 smoothly with transitions generated automatically.

In the literature, set operators include:
(1). Pure union and intersection [10]: $B_k=Max(x_1,...,x_k)$ and $Min(x_1,...,x_k)$, containing C^0 continuity only;
(2). Soft blends[2], union $B_k=x_1+x_2+...+x_k$;
(3). Super-ellipsoidal blends [10]: $B_k=(x_1^n+...+x^n)^{1/n}$, where n is a curvature parameter to adjust the shape of the transition;
(4). Perlin's set blend [8]: union B_k=1-(1-x_1)*...*(1-x_k), intersection $B_k=x_1*x_2*...*x_k$ and difference $B_k=x_1*(1-x_2)*...*(1-x_k)$.

2.3. The scale method.

To obtain better shape and size control of the resulting transitions of blends, the scale method was proposed in [12] to develop C^1 continuous blends. In the scale method, given a base surface $H_k(x_1,...,x_k)$=0, an intersection blend is given by

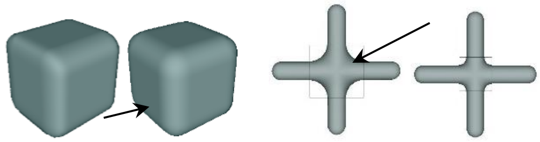
$$B_k(x_1,...,x_k)=\begin{cases} 0 & Min(x_1,\ldots,x_k)=0 \\ h_p & otherwise \end{cases},$$

where h_p is the root h of the equation $T(h)$=0, $T(h)=H_k(x_1/h-1, ...,x_k/h-1)$; and a union blend is given by

$$B_k(x_1,...,x_k)=\begin{cases} 0 & Max(x_1,\ldots,x_k)=0 \\ h_p & otherwise \end{cases},$$

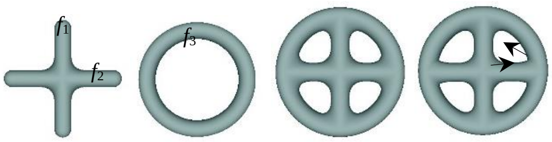
where h_p is the root h of the equation $T(h)$=0, $T(h)=-H_k(1-x_1/h, ...,1-x_k/h)$;

Furthermore, $H_k(x_1,...,x_k)= \sum_{i=1}^{k}[(r_i-x_i)/r_i]_+^{p_i}-1=0$ was applied as the base surface and so super-ellipsoidal union and intersection blends $B_k(f_1,...,f_k)$ were developed. These two blend have the following properties:
(1). They offer blending range parameters r_i and curvature parameters p_i, i=1,...,k, to adjust the shape and the size of the transition as shown in Figure 1.
(2).They can generate sequential blends with overlapping blending regions.
(3). They behave similar to $Max(f_1,...,f_k)$ and $Min(f_1,...,f_k)$ in non-blending regions, and hence deform primitives locally, as in Figure 1. However, this property unfortunately also lead to the following difficulty: in sequential blends such as $B_2(B_k(f_1,...,f_k), f_{k+1})$=0.5 with range parameters r_a and r_b for B_k and f_{k+1} in B_2, primitives $f_1,\ldots,f_k$ always have the same blending range r_a to blend with f_{k+1}. Consequently, they have similar subsequent blending surfaces with f_{k+1}, as shown in Figure 2(b). This means that blends $B_k(f_1,...,f_k)$ created from the scale method cannot respectively adjust their primitives' subsequent blends with f_{k+1} in sequential blends like $B_2(B_k(f_1,...,f_k), f_{k+1})$=0.5 and keep the shape of $B_k(f_1,\ldots,f_k)$=0.5 unchanged.



(a) (b)

Figure 1. (a) Intersection blends of six planes have different edges by setting planes different curvatures. (b) Union blends of two cylinders have different sizes of transitions, inside the box, by setting different blending ranges to primitives. All the primitives in (a) and (b) deform locally.



(a) (b) (c)

Figure 2. (a) Left: A union from the scale method $B_2(f_1, f_2)=0.5$ of cylinders; Right: A toroid $f_3=0.5$. (b). Sequential unions $B_2(B_2(f_1,f_2), f_3)=0.5$, where f_1 and f_2 always have similar subsequent blends with the toroid f_3. (c). Sequential unions $B_{S2}(B_{S2}(f_1,f_2), f_3)=0.5$. Because B_{S2} is a union blend from Section 3 and so f_1 and f_2 have different subsequent blends with f_3, pointed by arrows, by assigning f_1 and f_2 different values of subsequent blending range parameter.

To solve the difficulty stated above, Section 3 presents the non-uniform scaling method to develop new blends that offer each primitive an individual subsequent blending range parameter.

3. The non-uniform scaling method

This section presents the non-uniform scaling method to conquer the difficulty of the scale method stated in Section 2.3.

3.1. Steps of the non-uniform scaling method

This method is to develop a new blend that gives an individual subsequent blending range parameter to each primitive for controlling primitives' subsequent blends when the blend is used as a new primitive in sequential blends for soft object modeling. This method includes two steps as follows:

Step (1): Obtain a base surface $H_k(x_1,...,x_k)=0$ that is an existing union blending operator $H_k(x_1,\ldots,x_k)=0$ on $f_i(x,y,z)=0$, with blending range parameters r_i, $i=1,...,k$. Its shape is arc-shaped in its blending region and the same as $Min(x_1,\ldots,x_k)=0$ in non-blending regions, like the shape of $H_2(x_1, x_2)=0$ displayed by the dotted curve in Figure 3.

Step (2): Based on the surface $H_k(x_1,...,x_k)=0$ in **Step (1),** an intersection operator $B_{Ak}:R^k\rightarrow R_+$ and a union operator $B_{Sk}:R^k\rightarrow R_+$, with a blending range parameter r_i and a

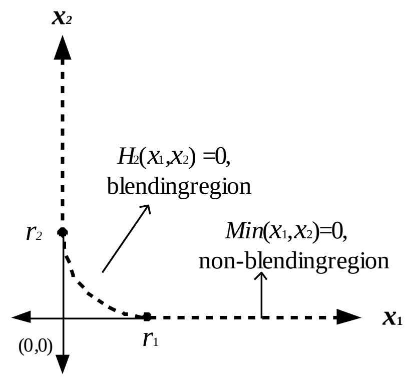


Figure 3. The shape of a two dimensional base curve $H_2(x_1, x_2)=0$.

subsequent blending range parameter m_i, $i=1,...,k$, for each primitive **,** are derived by:

(a). $$B_{Ak}(x_1,...,x_k)=\begin{cases} 0 & Min(x_1,\ldots,x_k)=0 \\ h_p & otherwise \end{cases} \quad (1)$$

where h_p is the root of equation $T(h)=0$ for a $(x_1,...,x_k)$ and

$$T(h)=H_k(\ x_1/(0.5^{(1-m1)}h^{m1})-1,\ ...,x_k/(0.5^{(1-mk)}h^{mk})-1\ ). \quad (2)$$

(b). $$B_{Sk}(x_1,...,x_k)=\begin{cases} 0 & Max(x_1,\ldots,x_k)=0 \\ h_p & otherwise \end{cases}, \quad (3)$$

where h_p is the root of equation $T(h)=0$ for a $(x_1,...,x_k)$ and

$$T(h)=-H_k(\ 1-x_1/(0.5^{(1-m1)}h^{m1}),\ ...,1-x_k/(0.5^{(1-mk)}h^{mk})\ ); \quad (4)$$

In the above, $r_i\leq 1$ are required for $i=1,...,k$, in Eq. (4) and $m_i>0$ for $i=1,...,k$, in Eqs. (2) and (4).

In fact, $B_{Ak}(x_1,...,x_k)=h$ and $B_{Sk}(x_1,...,x_k)=h$ have shapes similar to the surfaces

$$H_k(x_1-1,...,x_k-1)=0 \text{ and } -H_k(1-x_1,...,1-x_k)=0$$

which is performed non-uniform scaling via scaling factors:

$$[0.5^{(1-m1)}h^{m1},\ \ldots,\ 0.5^{(1-mk)}h^{mk}].$$

Therefore, B_{Ak} and B_{Sk} in Eqs. (1) and (3) have the following properties:

(1). No matter what positive values m_1,..., and m_k, are assigned, the shapes of intersection $B_{Ak}(f_1,...,f_k)=0.5$ and union $B_{Sk}(f_1,...,f_k)=0.5$ keep unchanged. In addition, this two blends always have blending ranges $r_i/2$, $i=1,\ldots,k$, for primitive f_1,..., and f_k and generate local blends without deforming primitives totally if $r_i\leq 1$ holds for $i=1,\ldots,k$.

This is proved by substituting 0.5 for h_p in Eqs. (2) and (4), which yields that $B_{Ak}(x_1,...,x_k)=0.5$ and $B_{Sk}(x_1,...,x_k)=0.5$ are always the same as

$$H_k(x_1/0.5-1,...,x_k/0.5-1)=0 \text{ and } -H_k(1-x_1/0.5,...,1-x_k/0.5)=0.$$

This means that $B_{Ak}(x_1,...,x_k)=0.5$ and $B_{Sk}(x_1,...,x_k)=0.5$ are always similar to $H_k(x_1-1,...,x_k-1)=0$ and $-H_k(1-x_1,...,1-x_k)=0$ scaled by [0.5, …, 0.5] whatever values m_1,..., and m_k, are set. The shapes of two-dimensional intersection and union operators $B_{A2}(x_1,x_2)=0.5$ and $B_{S2}(x_1,x_2)=0.5$ are displayed in blue and green curves in Figure 4.

(2). As displayed in blue and green curves in Figure 4, in non-blending regions $B_{Ak}(x_1,...,x_k)$ and $B_{Sk}(x_1,...,x_k)$ behave respectively like

$$Min(0.5(x_1/0.5)^{(1/m1)},...,0.5(x_k/0.5)^{(1/mk)}) \text{ and}$$

$$Max(0.5(x_1/0.5)^{(1/m1)},...,0.5(x_k/0.5)^{(1/mk)}). \qquad \textbf{(5)}$$

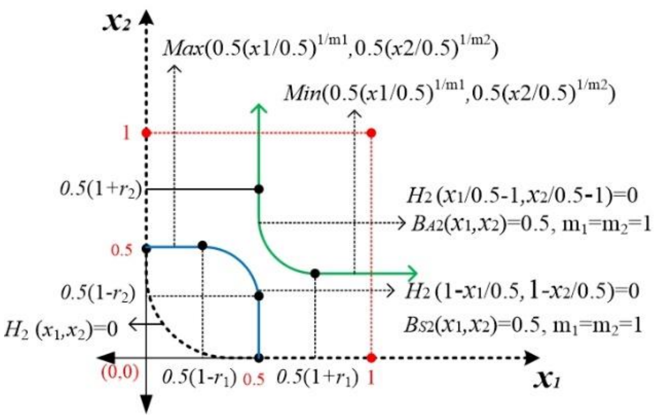


Figure 4. Blending operator curves of $B_{A2}(x_1, x_2)=0.5$ (green), and $B_{S2}(x_1, x_2)=0.5$ (blue) remain unchanged whatever values m_1,..., and m_k, are set.

This implies that the shapes of $B_{Ak}(x_1,...,x_k)=0.5$ and $B_{Sk}(x_1,...,x_k)=0.5$ in non-blending regions are always the same as $Min(x_1,...,x_k)=0.5$ and $Max(x_1,...,x_k)=0.5$ whatever values m_1,..., and m_k are set . Eq. (5) are obtained by solving the roots h of equations:

$$Min(\ x_1/(0.5^{(1-m1)}h^{m1})-1,\ ...,x_k/(0.5^{(1-mk)}h^{mk})-1\ )=0 \text{ and}$$

$$Max(\ x_1/(0.5^{(1-m1)}h^{m1})-1,\ ...,x_k/(0.5^{(1-mk)}h^{mk})-1\ )=0.$$

Eq. (5) tells that primitives f_i, i= 1 to k, become $0.5(f_i/0.5)^{(1/mi)}$ after blending $B_{Ak}(f_1,...,f_k)$ and $B_{Sk}(f_1,..., f_k)$. As a result, among level surfaces $B_{Ak}(f_1,...,f_k)=h$ or $B_{Sk}(f_1,...,f_k)=h$, $h\in R$, if $m_i>m_j$, level surface $f_i=h$ will dilate farther and more quickly than $f_j=h$ does, as shown in Figure 5. That is, if $m_i>m_j$, f_i will have a larger subsequent blending surface than f_i does when $B_{Ak}(f_1,...,f_k)$ and $B_{Sk}(f_1,..., f_k)$ are reapplied as new primitives in sequential blend. This solves the difficulty of the scale method stated in Section 2.3 by varying m_1,..., and m_k to adjust primitives' subsequent blending surfaces. This is shown in Figure 6, where the subsequent blending surface of f_1 gets larger and larger, as m_1 for f_1 in B_{A3} is increased from 0.3, 0.65, 1, and 1.5 for the objects from top left to bottom right.

(3). In sequential blends $B_{S2}(B_{Ak}(f_1,...,f_k),f_{k+1})=0.5$ with blending ranges r_a for $B_{Ak}(f_1,...,f_k)$ and r_b for f_{k+1} in B_{S2}, varying m_i makes primitives f_i, i=1,...,k, in B_{Ak} $(f_1,...,f_k)$ have different blending ranges by

$$(1-(1-r_a)^{mi})/2,\ i=1,...,k,$$

with f_{k+1} in B_{S2}. Especially, if $m_i>1$, the blending range of f_i with f_{k+1} is larger than $r_a/2$; if $m_i<1$, the blending range of f_i with f_{k+1} is less than $r_a/2$.

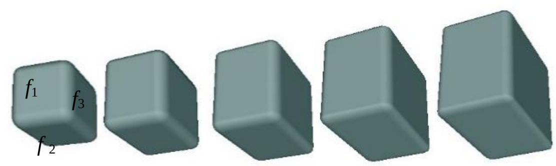


Figure 5. Level blending surfaces of an intersection on three parallel planes, by $B_{A3}(f_1,f_2,f_3)$=h with h=0.5, 0.45, 0.4, 0.35 and 0.3 for the objects from left to right. Due to m_1=0.6 and m_2=2.8, f_2 dilates farther and more quickly than f_1 does.

Similarly, in $B_{A2}(B_{Sk}(f_1,...,f_k),f_{k+1})=0.5$ with blending ranges r_a for B_{Sk} in B_{A2}, varying m_i makes primitives f_i, i=1,...,k, in $B_{Sk}(f_1,...,f_k)$ have different blending ranges by

$$((1+r_a)^{mi}-1)/2,\ i=1,...,k,$$

with f_{k+1} in B_{A2}. This means that if $m_i>1$, the blending range of f_i with f_{k+1} is larger than $r_a/2$; if $m_i<1$, the blending range of f_i with f_{k+1} is less than $r_a/2$.

That is, as m_i increases from 0 to ∞, the subsequent blending surface of f_i with f_{k+1} gets larger and larger.

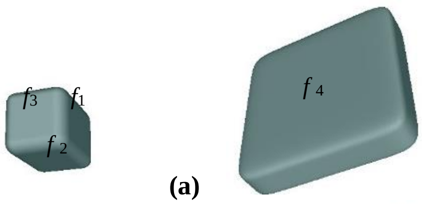
3.2. Calculating the gradients of B_{Ak} and B_{Sk}

Calculating the gradients of B_{Ak} and B_{Sk} is usually required in a shading process, and hence it is described below. Let both equations $T(h)=0$ in Eqs. (2) and (4) be viewed and written as equations of variables h, x_1,…, and x_k, by

$$T(h)\ \equiv\ G(h, x_1,..., x_k)=0.$$

Then, from the implicit theorem, both the gradients of $B_{Ak}(x_1,…,x_k)$ and $B_{Sk}(x_1,…,x_k)$ in Eqs. (1) and (3) are calculated through the values of root h_p, x_1, ... and x_k, respectively, by:

$$B_{Ak}^{(x_i)}(x_1,\dots,x_k)= -G^{(xi)}(h_p, x_1,..., x_k)/G^{(h)}(h_p, x_1,..., x_k),$$



(a)

(b)

Figure 6. (a). Left: An intersection $B_{A3}(f_1,f_2,f_3)=0.5$ on 3 pairs of parallel planes; Right: A super-ellipsoid $f_4=0.5$. (b). Unions of the objects in (a) by $B_{S2}(B_{A3}(f_1,f_2,f_3), f_4)=0.5$, where only the subsequent blending surface, of f_1 with f_4 is enlarged gradually but the one of f_2 with f_4, pointed by a dotted arrow, remains unchanged, because m_2 for f_2 is always 2.8 but m_1 for f_1 in B_{A3} is increased from 0.3, 0.65, 1 and 1.5 for the objects from top left to bottom right.

$i=1,\ldots,k$, where $G(h, x_1,\ldots,x_k)$ is $T(h)$ from Eq. (2);

$$B_{Sk}^{(x_i)}(x_1,\ldots,x_k)=-G^{(xi)}(h_p, x_1,\ldots, x_k)/G^{(h)}(h_p, x_1,\ldots, x_k),$$

$i=1,\ldots,k$, where $G(h, x_1,\ldots,x_k)$ is $T(h)$ from Eq. (4).

4. Blending operators with C^1 continuity

Based on the method in Section 3, two and high-dimensional B_{Ak} and B_{Sk} with C^1 continuity are derived and presented in this section.

4.1. Binary blending operators

In fact, 2D $H_2(x_1, x_2)=0$ in **Step (1)** can also be defined piecewise by the union of the curve $Min(x_1, x_2)=0$ and an arc-shaped curve tangent to $Min(x_1, x_2)=0$. Thus, let 2D base curve $H_2(x_1, x_2)=0$ be given by

$$H_2(x_1,x_2)=0$$

$$\equiv \begin{cases} H_H(x1,x2)=0 & \text{Blending region} \\ Min(x1,x2)=0 & \text{Non}-\text{blending region} \end{cases}$$

where $H_H(x_1,x_2)=r_2^2x_1^2+r_1^2x_2^2+r_2^2r_1^2-2r_2^2r_1x_1-2r_1^2r_2x_2+2px_1x_2$, $-\infty<p<r_1r_2$.

Then, conic blending operators B_{A2} and B_{S2}, possessing blend range parameters r_1 and r_2 and subsequent blend range parameter m_1 and m_2, for binary intersection and union blends are given, respectively, by:

(a). $B_{A2}(x_1, x_2)=$ **(6)**

$$\begin{cases} \left(\frac{x_1}{0.5^{(1-m1)}}\right)^{\left(\frac{1}{m1}\right)} & x_2 \ge (1+r_2)0.5^{(1-m2)}\left(\frac{x_1}{0.5^{(1-m1)}}\right)^{\left(\frac{m2}{m1}\right)} \\ \left(\frac{x_2}{0.5^{(1-m2)}}\right)^{\left(\frac{1}{m2}\right)} & x_1 \ge (1+r_1)0.5^{(1-m1)}\left(\frac{x_2}{0.5^{(1-m2)}}\right)^{\left(\frac{m1}{m2}\right)} \\ h_p & \text{otherwise} \end{cases}$$

where $m_1>0$, $m_2>0$, $h_p\in T^{-1}(0)$, and

$$T(h)=H_H(x_1/(0.5^{(1-m1)}h^{m1})-1, x_k/(0.5^{(1-m2)}h^{m2})-1);$$

(b). $B_{S2}(x_1, x_2)=$ **(7)**

$$\begin{cases} \left(\frac{x_1}{0.5^{(1-m1)}}\right)^{\left(\frac{1}{m1}\right)} & x_2 \ge (1-r_2)0.5^{(1-m2)}\left(\frac{x_1}{0.5^{(1-m1)}}\right)^{\left(\frac{m2}{m1}\right)} \\ \left(\frac{x_2}{0.5^{(1-m2)}}\right)^{\left(\frac{1}{m2}\right)} & x_1 \ge (1-r_1)0.5^{(1-m1)}\left(\frac{x_2}{0.5^{(1-m2)}}\right)^{\left(\frac{m1}{m2}\right)} \\ h_p & \text{otherwise} \end{cases}$$

where $m_1>0$, $m_2>0$, $r_1\le1$, $r_2\le1$, $h_p\in T^{-1}(0)$, and

$$T(h)=-H_H(1-x_1/(0.5^{(1-m1)}h^{m1}), 1-x_k/(0.5^{(1-m2)}h^{m2})).$$

To solve the root h_p of the equation $T(h)=0$ in Eqs. **(6)-(7)**, numerical methods: Newton-Ralphson method is applied by using

$$h=Min((x_1/0.5^{(1-m1)})^{1/m1}, (x_2/0.5^{(1-m2)})^{1/m2})$$

as the initial guess for Eq. (6), and by using

$$h=Max((x_1/0.5^{(1-m1)})^{1/m1}, (x_2/0.5^{(1-m2)})^{1/m2})$$

as the initial guess for Eq. (7).

4.2. High-dimensional blending operators

Let base surface $H_k(x_1,\ldots,x_k)=0$ in Section 3.1 be given by super-ellipsoids

$$H_k(x_1,...,x_k)=\sum_{i=1}^{k}[(r_i - x_i)/r_i]_+^{p_i} - 1 = 0 .$$

Then, from Eqs. (1) and (3), super-ellipsoidal intersection and union operators B_{Ak} and B_{Sk}, with blend range parameters r_1,..., and r_k, and curvature parameters p_1,..., and p_k, and subsequent blend range parameters m_1,..., and m_k, on soft objects are given, respectively, by

(a). $$B_{Ak}(x_1,...,x_k)=\begin{cases} 0 & Min(x_1,\dots,x_k)=0 \\ h_p & \text{otherwise} \end{cases} \quad \textbf{(8)}$$

where $m_i>0$ must hold for $i=1,...,k$, h_p is the root of equation $T(h)=0$ for a point $(x_1,...,x_k)$, and

$$T(h)=\sum_{i=1}^{k}[(r_i - x_i/(0.5^{(1-m_i)}h^{m_i})+1)/r_i]_+^{p_i} - 1 ;$$

(b). $$B_{Sk}(x_1,...,x_k)=\begin{cases} 0 & Max(x_1,\dots,x_k)=0 \\ h_p & \text{otherwise} \end{cases} , \quad \textbf{(9)}$$

where $m_i>0$ and $r_i \leq 1$ must hold for $i=1,...,k$, h_p is the root of equation $T(h)=0$ for any point $(x_1,...,x_k)$, and

$$T(h)=1-\sum_{i=1}^{k}[(r_i + x_i/(0.5^{(1-m_i)}h^{m_i})-1)/r_i]_+^{p_i} .$$

The roots h_p of equation $T(h)=0$ in Eqs. **(8)-(9)** are solved by Newton-Ralphson method and using the value

$$h=Min(\ (x_1/0.5^{(1-m1)})^{(1/m1)},...,\ (x_k/0.5^{(1-mk)})^{(1/mk)}\ )$$

as the initial guess for Eq. (8), and using the value

$$h=Max(\ (x_1/0.5^{(1-m1)})^{(1/m1)},...,\ (x_k/0.5^{(1-mk)})^{(1/mk)}\ )$$

as the initial guess for Eq. (9).

5. Conclusions

In soft object modeling, existing blends do not have an individual blending range control on each primitive's subsequent blend when they are used as a new primitive in other blends, i.e., sequential blends. As a result, their primitives always have similar subsequent blending surfaces. In order to solve the above difficulty, this paper has proposed the non-uniform scaling method. This method can transform an existing union blend into a new blend that additionally offer each primitive a subsequent blending range parameter for adjusting the primitive's subsequent blend when the blend is used as a new primitive in other blends. Thus, all their primitives have different subsequent blending surfaces by varying primitives' subsequent blending range parameters. From this method, elliptic and super-ellipsoidal union and intersection blends have been created successfully for soft object modeling. These newly developed blends:

(1). Have respective blending range controls (respective subsequent blending range parameters) on every primitive's subsequent blend and whatever values these parameters are set, the original blending surface always keep unchanged while reused as a new primitive in other blends.

(2). Provide each primitive a blending range parameter and a curvature parameter to adjust the size and the shape of the transition of each primitive's blending surface.

(3). Are C^1 continuous everywhere and hence can generate smooth sequential blends containing blending regions overlapped.

(4). Are capable of deforming primitives locally after blending because they behave like pure union and intersection blends in non-blending regions.

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